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THE ANALYSIS AND DESIGN OF COMMUNICATION  
NETS USING RELIABILITY FUNCTIONS

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THE ANALYSIS AND DESIGN OF  
COMMUNICATION NETS  
USING RELIABILITY FUNCTIONS

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# ABSTRACT

Topological methods may be employed in an effective manner in the analysis and design of a communication net. One application is in the determination of the reliability of the net with respect to both complete communication and k-terminal communication. The effect on this reliability caused by the removal of one link is studied.

The k-terminal reliability function is defined and its application to analysis and design of communication nets is demonstrated. A digital computer program is presented, and examples of its use are included. A formula is given to find the value of the 2-terminal function.

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## 1. Introduction.

Any system that provides for communication, be it the transmission of information or the transportation of commodities, is subject to possible failure. The loss of security, the malfunction of a radio transmitter, and the destruction of a bridge are examples of this failure. Linear graph theory, as described in texts such as [5], has been applied to measure the reliability of the system given some measure of the reliability of the parts. Fu and Yau, [4], have indicated a value for the probability that two stations can communicate; and a digital computer algorithm for this has been given, [6]. The probability that all pairs of stations can communicate within a given time interval has been studied by Chan, [2], and Fu, [3].

In this work the application of the  $k$ -terminal reliability function, to be defined in Section 3, is presented, and a digital computer program is included. In addition, the application of the power of this function without computer assistance is demonstrated. A formula is given to find the value of the 2-terminal reliability function without enumerating the Table of Combinations.

## 2. Statement of the Problem.

A communication net may be represented by a linear graph in which the edges of the graph represent the communication links, and the nodes of the graph represent the stations of the communication net.

A weight can be assigned to each edge of the graph to indicate the probability that the particular communication link represented by the edge will operate successfully. This weight is called the reliability of that edge. This probability is not a conditional probability. Each edge is assumed to be associated with an independent random variable, having only two possible states: normal operation, and not operational.

The communication net can be analyzed by studying the topology of the graph. Topological methods can be applied on several different problems. For example, if one link of a communication net has been lost (destroyed), it must be decided whether or not to replace this link with one of the other existing links in the net. The information needed to make this decision must include the relative importance of each link, some measure of the quality of the branch (e.g., its reliability), and the role of the branch in the net. The analysis of the data must include the degree of importance attached to the overall net reliability as contrasted to the reliability of communication between a pair or a group of terminals.

Several investigators, [3], [4], [6], have proposed quantitative measures for the reliability of a communication net. Investigations by Chan [2], have led to a result which uses the reliability function as a k-terminal reliability function. This function may be applied to the reliability of communication between any k-number of nodes. The topological methods which are used to generate the function are suitable

for use on a digital computer. The large storage requirement is the limiting factor in their application.

### 3. Properties of Reliability Functions.

#### 3.1 2-terminal Reliability Function.

The reliability function is defined in reference [2]. Some of the most important definitions are included here for ready reference and emphasis. Using the notation of [2], there is a value of reliability,  $p_1, p_2, \dots, p_e$ , assigned to each of the edges of the graph,  $e_1, e_2, \dots, e_k, \dots, e_e$ . Each branch of the net is assumed to be associated with an independent random variable. If the branch,  $b_i$ , is operational, the edge,  $e_i$ , of the graph assumes the value  $p_i$ , which is the probability that edge,  $e_i$ , will operate normally. If the branch is not operational, the edge,  $e_i$ , assumes the value  $\bar{p}_i$ , which is the probability that the edge,  $e_i$ , will not be operational. Although these reliabilities are probabilities, they are considered not to be conditional probabilities. That is, we have the following relations:

$$p_i + \bar{p}_i = 1 \quad (3.1)$$

$$\Pr (A \text{ and } B) = \Pr(A)\Pr(B) \quad (3.2)$$

#### Definition 3.1: 2-terminal Reliability Function:

A reliability function expressed in terms of the edge reliabilities of a network is a 2-terminal reliability function if it represents the reliability between two distinct terminals (i.e., the probability of communication between these two terminals).

Let  $q_{x,y}$  represent the 2-terminal reliability function between terminals  $x$  and  $y$ . Then we have

$$q_{x,y} = m_1 \bar{p}_1 \bar{p}_2 \dots \bar{p}_e + m_2 \bar{p}_1 \bar{p}_2 \dots \bar{p}_{e-1} p_e + \dots + m_e p_1 p_2 \dots p_e \quad (3.3)$$



where  $m_1, m_2, \dots, m_i, \dots, m_{2e}$  are either 1 or 0 depending on whether the combination of states of the edges provide a communication path between terminals  $\underline{x}$  and  $\underline{y}$ . If the path exists, then  $m_i = 1$ ; if not,  $m_i = 0$ . When the function is expressed as in equation (3.3), it is said to be in its canonical form.

Definition 3.2: Path Product.

A path set is a set of branches of a communication path between a pair of terminals. A path product is the product of the variables associated with the elements in a path set.

Definition 3.3: Path Product Term or Primary Path Product Term.

A canonical term is a path product term if

- (1) its unbarred variables are those of a path product; and
- (2) all other variables in the term are barred variables.

Definition 3.4: Secondary Path Product Term.

A canonical term is a secondary path product term if its unbarred variables correspond to those elements forming a path set plus one or more not in the path set.

Definition 3.5: Table of Combinations.

The collection of all nonzero canonical terms of a 2-terminal reliability function is called the Table of Combinations. That is, all the terms in which  $m_i$  takes on the value 1.

Definition 3.6: k-edged Path.

A k-edged path is a path of  $\underline{k}$  number of edges or branches which constitute a communication path between a pair of terminals.

Theorem 3.1:

Let  $P_A$  be an  $A$ -edged path between terminals  $\underline{x}$  and  $\underline{y}$ , in a graph containing  $e$  number of edges. The Table of Combinations for the 2-terminal reliability function,  $q_{x,y}$ , will include a primary path product term for path  $P_A$ , and  $2^{(e-A)} - 1$  secondary path terms.

Proof:

From definition (3.3) and (3.5), the primary path product term must be present. There are  $2^{(e-A)}$  possible combinations of the  $(e-A)$  variables not specified by the primary term. However, definition (3.4) requires the number of unbarred variables to be at least  $(A+1)$ . Therefore, the maximum possible number of combinations is  $2^{(e-A)} - 1$ .

If a graph contains  $e$  number of edges and  $n$  number of vertices, then definition (3.1) indicates that there will be  $2^e$  number of terms in the 2-terminal reliability function. Theorem (3.1) indicates that one path of  $A$  number of edges will produce  $2^{(e-A)}$  terms in the Table of Combinations. This is the same as saying that, from definition (3.1),  $m_i$  will be nonzero in  $2^{(e-A)}$  terms of the 2-terminal reliability function.

Lemma 3.1:

If no path exists between two terminals  $x$  and  $y$ , then the 2-terminal reliability function will have no nonzero terms.

Proof:

By definition (3.1) all the  $m_i$  values will be zero, since there is no path between terminals  $x$  and  $y$ . There will be no entries in the Table of Combinations.



Theorem 3.2:

If a graph contains  $e$  number of edges and  $n$  number of vertices, and if an  $A$ -edged path is the only path between vertices  $x$  and  $y$ , then there will be  $2^{(e-A)}$  number of terms in the 2-terminal reliability function,  $q_{x,y}$ .

Proof:

By theorem (3.1) there will be  $2^{(e-A)}$  number of terms for the  $A$ -edged path. Since there are no more paths, lemma (3.1) shows there will be no additional terms. Hence, the theorem.

Theorem 3.3:

Let  $P_A$  be an  $A$ -edged path and  $P_B$  a  $B$ -edged path, each between terminals  $x$  and  $y$ , in a graph containing  $e$  number of edges. The primary path product term of path set  $A$  can not be a secondary path product term for path set  $B$  in  $q_{x,y}$ .

Proof:

The unbarred elements in the secondary path product terms must appear as barred elements in the primary path product term of the path set  $A$ , by definition (3.3). Hence, the theorem.

Theorem 3.4:

Let  $P_A$  and  $P_B$  be path sets between terminals  $x$  and  $y$ . Then at least one secondary path product term for path set  $A$  will be identical to one for path set  $B$ .

Proof:

From definition (3.4), the collection of secondary path product terms for any path set will include the case where all variables are unbarred.

This will be common to all collections of secondary path product terms.

Hence, there will always be this term present.

Example 3.1:

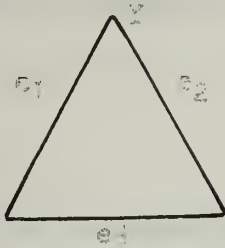


Figure 3.1

$$\text{Path Set A} \quad P_A = \{e_1\}$$

$$\text{Path Set B} \quad P_B = \{e_2 e_3\}$$

2-terminal reliability function,  $q_{x,y}$ , expressed in canonical form:

$$q_{x,y} = m_1 \bar{p}_1 \bar{p}_2 \bar{p}_3 + \quad (7) \quad (10)$$

$$m_2 \bar{p}_1 \bar{p}_2 p_3 + \quad (8)$$

$$m_3 \bar{p}_1 p_2 \bar{p}_3 + \quad (9)$$

$$m_4 \bar{p}_1 p_2 p_3 + \quad (2) \quad (11)$$

$$m_5 p_1 \bar{p}_2 \bar{p}_3 + \quad (1)$$

$$m_6 p_1 \bar{p}_2 p_3 + \quad (3) \quad (12)$$

$$m_7 p_1 p_2 \bar{p}_3 + \quad (4) \quad (13)$$

$$m_8 p_1 p_2 p_3 \quad (5) \quad (6) \quad (14)$$

(1) primary path product term for path set A

(2) primary path product term for path set B

(3)(4)(5) secondary path product terms for path set A

(6) secondary path product term for path set B

observe that (5) and (6) are identical

(7)(8)(9)  $m_i$  terms are 0 because terms do not contain a path between terminals x and y

(10) this term will always be zero

(11)(12)(13) a tree of the graph is made up of the unbarred terms

(14) a tree plus a chord of the graph is made up by the unbarred terms

### 3.2 Multi-terminal Reliability Function.

Reliability functions, which are expressed in terms of the edge reliabilities, may be derived in order to represent the reliability of transmission between more than two terminals. Thinking of a communication net, one can envision a classification of the terminals into the categories of transmitter, receiver, or both, and repeaters. The repeaters may be looked upon as intermediate steps in the communication paths. All of this may enter into an evaluation of the reliability of simultaneous communication between all pairs of terminals.

#### Definition 3.7: Complete Communication.

The event that every pair of terminals  $x$  and  $y$  in a net can communicate with each other simultaneously is complete communication; the simultaneous communication between all pairs of terminals of the net.

#### Definition 3.8: N-terminal Reliability.

The probability of complete communication in a net with  $N$ -terminals is called the "N-terminal reliability" of the net and is denoted by  $q_n$ .

#### Definition 3.9: k-terminal Reliability.

In a set of  $n$  number of terminals, consider  $k$  number of terminals of transmission, where  $k < n$ . Then, the probability of complete communication among the set of  $k$  number of terminals is called the  $k$ -terminal reliability of the net and is denoted by  $q_k$ .

From definitions (3.7) and (3.9), a net may have several different values of  $k$ -terminal reliability depending on which terminals are included in the set of  $k$  number of terminals of transmission. The

choice of terminals included in this set is of fundamental importance in the interpretation of the analysis or data collection.

For a given net,  $q_n$  is found in the following way: first determine  $q_{x,y}$  for each pair of vertices. There will be  $\frac{1}{2}n(n-1)$  for a net with  $n$  number of terminals. Then sum all the canonical terms which are common to all  $q_{x,y}$  functions. To find  $q_k$ , find  $q_{x,y}$  for each pair of terminals in the set,  $K$ . There will be  $\frac{1}{2}k(k-1)$  number. Then sum all the canonical terms common to all these  $q_{x,y}$  functions.

Another method is first to determine a complete set of trees for the net. Complete communication is possible if and only if the unbarred variables of a canonical term include those corresponding to the branches of a tree. By taking the sum of all canonical terms that contain a tree, or a tree plus chords, made up from the unbarred variables,  $q_n$  is determined. If the terms which are summed contain a subtree of the graph, or a subtree plus chords, containing all the vertices of the set  $K$ , the  $q_k$  is determined. This method can be applied on the canonical form resulting after the first  $q_{x,y}$  calculation. This can reduce the labor required to find  $q_n$  or  $q_k$  provided all the trees of the graph are known.

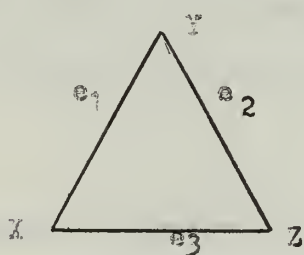


Figure 3.1 repeated

$$\text{Path set } P_C = \{e_2\}$$

$$\text{Path set } P_D = \{e_1 e_3\}$$

$$\text{Path set } P_E = \{e_3\}$$

$$\text{Path set } P_F = \{e_1 e_2\}$$

#### Part A

$$\begin{aligned} q_{y,z} = & m_1 \bar{p}_1 \bar{p}_2 \bar{p}_3 + \\ & m_2 \bar{p}_1 \bar{p}_2 p_3 + \\ & m_3 \bar{p}_1 p_2 \bar{p}_3 + (1) \\ & m_4 \bar{p}_1 p_2 p_3 + (3) (7) \end{aligned}$$

#### Part B

$$\begin{aligned} q_{x,z} = & m_1 \bar{p}_1 \bar{p}_2 \bar{p}_3 + \\ & m_2 \bar{p}_1 \bar{p}_2 p_3 + (1) \\ & m_3 \bar{p}_1 p_2 \bar{p}_3 + \\ & m_4 \bar{p}_1 p_2 p_3 + (3) (7) \end{aligned}$$

$$\begin{aligned}
& m_5 p_1 \bar{p}_2 \bar{p}_3 + \\
& m_6 p_1 \bar{p}_2 p_3 + (2) (8) \\
& m_7 p_1 p_2 \bar{p}_3 + (4) (9) \\
& m_8 p_1 p_2 p_3 \quad (5) (6) (10)
\end{aligned}$$

$$\begin{aligned}
& m_5 p_1 \bar{p}_2 \bar{p}_3 + \\
& m_6 p_1 \bar{p}_2 p_3 + (4) (8) \\
& m_7 p_1 p_2 \bar{p}_3 + (2) (9) \\
& m_8 p_1 p_2 p_3 \quad (5) (6) (10)
\end{aligned}$$

- (1) Primary path product term for path set C and E.  
 (2) Primary path product term for path set D and F.  
 (3)(4)(5) Secondary path product term for path set C and E.  
 (6) Secondary path product term for path set D and F.  
 (7) (8)(9) (10) make up a tree, or a tree plus a chord, of the graph, made up of the unbarred terms.

### Example 3.3



Figure 3.1 repeated

The 3 trees of the graph are:

$$\{e_1 e_2\}, \{e_2 e_3\}, \{e_1 e_3\}$$

Using the results of examples (3.2) and (3.1), we have

$$\begin{aligned}
q_n = & m_4 \bar{p}_1 p_2 p_3 + \\
& m_6 p_1 \bar{p}_2 p_3 + \\
& m_7 p_1 p_2 \bar{p}_3 + \\
& m_8 p_1 p_2 p_3
\end{aligned}$$

by the process of summing the canonical terms in common. From example (3.1) alone, or (3.2) part (a) alone or part (b) alone, the same results

Number of trees,  $t$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = 3$$



could have been obtained using the method of trees.

In this simple example, for any set of terminals of transmission less than the given set, the problem reduces to one of the examples, since it becomes a 2-terminal reliability problem.

### 3.2 Some Characteristics of the Path Product Terms.

A graph contains 5 edges. A matrix can be constructed in which the elements are the number of path product terms that contain U number of unbarred variables compared to the L number of edges in the path.

Path Length	Number of unbarred variables				
	1	2	3	4	5
4	0	0	0	1	1
3	0	0	1	2	1
2	0	1	3	3	1
1	1	4	6	4	1

The number of path product terms

This array is recognized as the start of Pascal's Triangle. See reference [1]. The nonzero matrix elements are the so called binary coefficients, defined by the formula

$$\binom{n}{j} = \frac{n!}{j!(n-j)!} \quad (3.4)$$

where, in this case,  $n = (e-A)$ , and A is the path length, and  $j = 0, 1, 2, \dots, (e-A)$ .

If all the branches of the net have the same reliability,  $p$ , then the collection of all path product terms resulting from an  $A$ -edged path, in a graph with  $e$  edges, can be written as follows:

$$\begin{aligned}
& \binom{e-A}{0}_p p^A \bar{p}^{(e-A)} + \binom{e-A}{1}_p p^{(A+1)} \bar{p}^{(e-A-1)} + \binom{e-A}{2}_p p^{(A+2)} \bar{p}^{(e-A-2)} + \\
& \dots + \binom{e-A}{e-A-1}_p p^{(e-1)} \bar{p}^{(1)} + \binom{e-A}{e-A}_p p^e = \\
& \sum_{i=0}^{e-A} K(i)_p p^{(A+i)} \bar{p}^{(e-A-i)}
\end{aligned} \tag{3.5}$$

where  $K(i)$  is the collection of the binary coefficients defined by the formula

$$\binom{e-A}{i} = \frac{n!}{i!(n-i)!}$$

Examples (3.1) and (3.2) illustrate the generation of the primary and secondary path product terms. Theorem (3.4) points out that there will always be at least one of the secondary path product terms common to the expansions generated from two or more path sets is a 2-terminal reliability function. Theorem (3.3) proves that the primary path product term is never a secondary path product term for another path in the 2-terminal reliability function; that is, it is not one of the duplicate terms. The exact number of unique terms added to this function is a problem in conditional probability. The following group of theorems is fundamental to the solution of this problem.

The theorems and lemmata in this section are subjected to the following conditions: a graph contains e number of edges and n number of vertices. Path set,  $P_{1, xy}$ , is a 1-edged path set between terminals x and y, made up of branch b, alone.

Theorem 3.5:

No multi-edged path between terminals  $x$  and  $y$  may contain branch  $\underline{b}$ .

Proof:

By definition (3.2) a path set is a set of branches of a communication path between a pair of terminals. No additional branches are included in the path set. Hence, since branch  $\underline{b}$  is a path by itself, any set that is made up of branch  $\underline{b}$  and one or more additional branches cannot be a path set between terminals  $\underline{x}$  and  $\underline{y}$ . Definition (3.6) of a  $k$ -edged path allows the conclusion that no multi-edged path between terminals  $\underline{x}$  and  $\underline{y}$  contain branch  $\underline{b}$ .

Lemma 3.2:

There can be no primary path product term in which branch  $\underline{b}$  appears as an unbarred variable in the 2-terminal reliability function,  $q_{x,y}$ , except path set  $P_{1,xy}$ .

Proof:

Lemma (3.2) follows immediately from theorem (3.5) and definition (3.3).

Lemma 3.3:

There can be no secondary path product terms generated from any path between  $\underline{x}$  and  $\underline{y}$  in which branch  $\underline{b}$  appears as an unbarred variable except path set  $P_{1,xy}$ .

Proof:

This lemma follows directly from theorem (3.5) and definition (3.4).



Theorem 3.6:

Let path set,  $P_{2,xy}$ , be an A-edged path set between terminals  $\underline{x}$  and  $\underline{y}$ . This path will generate  $2^{(e-A-1)}$  secondary terms that are duplicates of those generated by  $P_{1,xy}$ .

Proof:

The number of possible unique combinations of the  $(e-A)$  number of barred variables has decreased by a factor of 2 because the 1-edged path,  $P_{1,xy}$ , has previously formed  $2^{(e-1)}$  combinations of the same variables. Only those in which the variable  $\underline{b}$  appears as a barred variable remain ungenerated. That number is  $2^{(e-A-1)}$ . There were  $2^{(e-A)}$  possible, but now only  $2^{(e-A-1)}$ , meaning that  $2^{(e-A-1)}$  were duplicates.

Investigations have shown that the number of duplicates generated among all the path product terms are arranged in correspondence with the binary coefficients. That is, if there are eight duplicated generated, there will be a 1-3-3-1 distribution, based on the number of unbarred variables in the path. If there are D number of duplicates, then the distribution will be as given by

$$\binom{D}{j} = \frac{D!}{j! (D-j)!} \quad , \quad j = 0, 1, 2, \dots, D$$

The number of unique path product terms that are generated by each path set,  $P_i$ , in the 2-terminal reliability function,  $q_{x,y}$ , can be found by the following method.

- a) Write the path matrix, order  $(p \times e)$ , where  $p$  is the number of path sets between terminals  $\underline{x}$  and  $\underline{y}$ , and  $e$  number of edges in the graph.
- b) Rearrange the rows such that the shortest path is row #1, the next shortest is row #2, and so forth, until the last

row contains the longest path.

- c) Count the number of columns in the row in which there is no entry in any of the preceeding rows. For example, this number will be  $(e-k)$  for the first row. This number may be zero. Call this number  $n_1$ .
- d) Calculate  $2^{n_1}$ . This result is the number of unique path product terms generated by the path set in the 2-terminal reliability function.

The total number of terms in the Table of Combinations is the sum of the results of step (d) above. It is seen that if  $n_1$  is zero, that one term is still added to the Table of Combinations by the path, which agrees with Theorem (3.3). If there is only one path, then there will be  $2^{(e-k)}$  number of terms which agrees with Theorem (3.2). Further statements which tend to prove the validity of this method follow the presentation of this example.

#### Example 3.4:

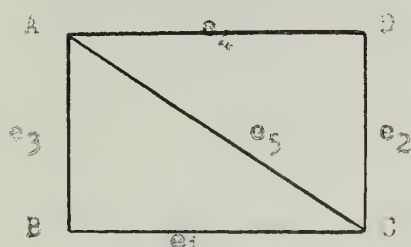


Figure 3.2

The paths between vertices A and B:

$$\{e_1 e_5\}, \{e_3\}, \{e_1 e_2 e_4\}$$

Path Matrix:

edges:	1	2	3	4	5
paths: 1	1	0	0	0	1
2	0	0	1	0	0
3	1	1	0	1	0

After rearranging, the path matrix becomes:

edges:		1	2	3	4	5
paths:	2	0	0	1	0	0
	1	1	0	0	0	1
	3	1	1	0	1	0

Counting the number of columns in each row in which there is no entry in that column in any of the preceeding rows:

		$\underline{n_i:}$
paths:	2	4
	1	2
	3	0

Calculating:

		$\underline{2^{n_i}:}$
paths:	2	16
	1	4
	3	1

From definitions (3.1 - 5), there are established conditions which a collection of variables must meet in order to be an entry in the Table of Combinations. Looking at example (3.4), one can see that path (2) will generate  $2^4$  unique terms. After they have been generated, path (1) generates a collection of  $2^3$  number of possible terms. All these in which edge (3) appears as an unbarred variable will be duplicates of those generated by path (1). This reduces the number of terms by a factor of 2, yielding the result:  $2^2$ . This can be understood from the concept of combinations; there were only 2 elements to be combined in 2 different ways vice what had appeared to be 3 elements; i.e.,  $2^2$  vice  $2^3$ .

The same argument holds for path (3). While there would appear to be 2 elements that can be varied, there are none which have not already been varied. The fact that the primary path product term is unique itself is given in theorem (3.3). It is from this logic that the above stated procedure is derived.

The knowledge of these numbers leads to a method to determine the value of the 2-terminal reliability function. With equation (3.5) in mind, a new equation can be written for each path set using the numbers,  $n_i$ , found by the above described procedure. Let  $P_{x,y}$  be a  $k$ -edged path between terminals  $x$  and  $y$  in a graph with  $e$  number of edges. For this path, let  $n_i = N$ . Then one can write

$$(r_i r_j \dots r_k) (\bar{r}_L \bar{r}_M \dots \bar{r}_N) \left\{ \binom{N}{0} \bar{r}^N + \binom{N}{1} r \bar{r}^{(N-1)} + \dots + \binom{N}{N-1} r^{(N-1)} \bar{r} + \binom{N}{N} r^N \right\} = |Q|_{P_k} \quad (3.6)$$

where  $|Q|_{P_k}$  is the value of all the path product terms added to the Table of Combinations by the path,  $P_k$ .

Where the first factor is the product of the  $k$  number of branch reliabilities associated with the branches in the path,  $P_{x,y}$ ; the second factor is the product of the  $(e-k-N)$  number of complements of the branch reliabilities of the branches which have previously appeared as unbarred variables in other path product terms; the third factor is the summation of all the path product terms in which  $N$  number of variables, associated with the branches not in the path,  $P_{x,y}$ , and not previously specified as being unbarred in other path product terms, appear as barred and unbarred elements.

However, the quantity inside the brackets is equal to one. This statement is true because the quantity inside the brackets is an application of the binomial theorem for two conditions: the branch reliabilities being all equal; and the branch reliabilities being not all equal.

Case 1) Let all the branch reliabilities be equal to  $r$ .

Then the quantity inside the brackets of equation (3.6) becomes

$$(r + \bar{r})^N$$

Since  $(r + \bar{r}) = 1$ , then  $(r + \bar{r})^N = 1$ .

Case 2) Let the reliability of branch,  $b_i$ , be given by  $r_i$ .

Then the quantity inside the brackets of equation (3.6) may be rewritten:

$$(r_1 + \bar{r}_1)(r_2 + \bar{r}_2) \dots (r_N + \bar{r}_N) = (1)(1)\dots(1) = 1$$

Therefore, equation (3.6) can be rewritten

$$(r_i r_j \dots r_k)(\bar{r}_L \bar{r}_M \dots \bar{r}_N) = \left| Q \right|_{P_k} \quad (3.7)$$

In the event all branch reliabilities are equal,

$$r^k \bar{r}^{(e-k-N)} = \left| Q \right|_{P_k} \quad (3.8)$$



#### 4. Topological Analysis of the Net.

##### 4.1 Collection of All Possible Path Sets.

Fundamental to the analysis of any communication net is the knowledge of all possible paths between the terminals  $x$  and  $y$ . Since no path may be ignored, even in relatively small, simple nets, the task of finding all paths is a laborous task. Linear graph theory provides an efficient method to accomplish this task.

Consider a graph with  $e$  number of edges and  $n$  number of vertices, and define a circuit to be a closed edge train. Then, the theory of topology has shown there to be  $(e - n + 1) = w$  number of independent circuits in the graph. These are called the fundamental circuit sets, and are denoted by  $B_f$ . Any additional circuits that appear to exist in the graph can be shown to be algebraically derived from the set of independent circuits.

Next there must be built a collection of sets, called set  $B$ , which includes the null set, the  $w$  number of sets of  $B_f$ , and the collection of all possible combinations of circuit sets, which are generated by the ringsum operation of two or more circuit sets in  $B_f$ . The ringsum operation is defined as modulo 2 addition of the respective column entries of the two sets. (See example, (4.2)). The set  $B$  will contain  $2^w$  number of sets.

A path set,  $P_{x,y}$ , is chosen, and expressed as a row matrix of  $e$  columns. Set  $B$  is represented in matrix form of order  $(w \times e)$ . The ringsum operation of  $P_{x,y}$  and each set in  $B$  produces a new set which is written in a collection of sets called set BRP. This is also in matrix form of order  $(w \times e)$ . Linear graph theory shows that the ringsum of a path set and a circuit set will be either a new path set or the disjoint

union of a path set and a circuit set. Now each entry in BRP must be examined to retain only the pure path sets, which are written in set PFC, also in matrix form of order  $(p \times e)$ , where  $p$  is the number of paths between  $\underline{x}$  and  $\underline{y}$ .

One process of examining set BRP is to look at each entry first to see whether the original path set,  $P_{x,y}$ , remains. If it does, and if there are other branches present, then it is not a pure path set and must be discarded. If it does not, then it must be ascertained whether any of the circuits of set B are included. If so, then it is not a pure path set. Those that remain are the pure path sets, and more importantly, all the path sets between terminals  $\underline{x}$  and  $\underline{y}$ . A block diagram of this procedure is given in figure (4.2).

The ringsum operation is built into the computer program by means of three FORTRAN IV instructions. The block diagram for this operation is shown in figure (4.3). Example (4.1) shows the ringsum operation for the four possible cases. The parameters must be logical elements; that is, either True, or False, with a 1 bit representing True, and a 0 bit False.

Example 4.1:

	<u>Case 1</u>	<u>Case 2</u>	<u>Case 3</u>	<u>Case 4</u>
A =	0	1	1	0
B =	1	0	1	0
	—	—	—	—
C =	1	1	0	0

The step-by-step computer program solution proceeds as follows:

after step 1, C =	0	0	0	0
after step 2, C =	1	1	1	0
after step 3, C =	1	1	0	0

The final results are identical.

Example 4.2:

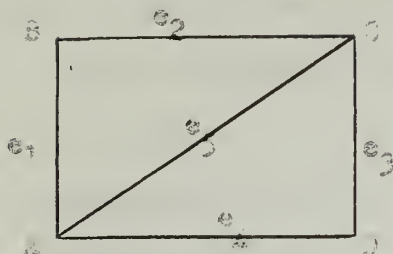


Figure 4.1

$$e = 5$$

$$m = 4$$

$$w = 5 - 4 + 1 = 2$$

$$2^w = 2^2 = 4$$

$$\text{Circuit (1)} \{e_1 e_2 e_5\}$$

$$\text{Circuit (2)} \{e_3 e_4 e_5\}$$

Set  $B_f$ :

edge number		1	2	3	4	5	
Circuit number	1	1	1	0	0	1	Order ( $w \times e = 2 \times 5$ )
	2	0	0	1	1	1	

Circuit (3) =  $\{e_1 e_2 e_3 e_4\}$  is derived algebraically by ringsum of circuit (1) and (2) as follows

1	1	0	0	1
0	0	1	1	1
<hr/>				
1	1	1	1	0

Choose  $P_{A,B} = \{e_1\}$

Set B:

edge number		1	2	3	4	5	
set number	1	0	0	0	0	0	Order ( $2^w \times e = 4 \times 5$ )
	2	1	1	0	0	1	
	3	0	0	1	1	1	
	4	1	1	1	1	0	

$$P_{AB} \oplus B = BRP$$



Set BRP =

edge number	1	2	3	4	5
set number	1	0	0	0	0
2	0	1	0	0	1
3	1	0	1	1	1
4	0	1	1	1	0

Order ( $2^w \times e = 4 \times 5$ )

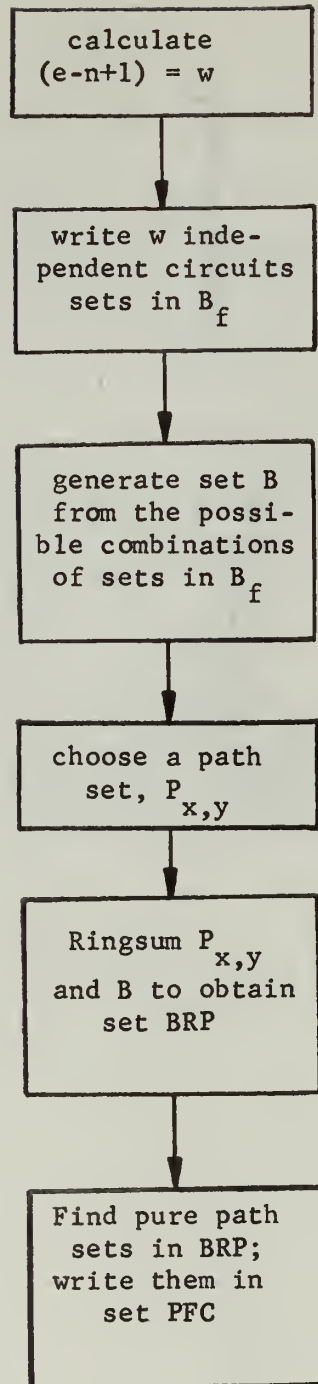
Set PFC =

edge number	1	2	3	4	5
path number	1	0	0	0	0
2	0	1	0	0	1
3	0	1	1	1	0

Order ( $p \times e = 3 \times 5$ )

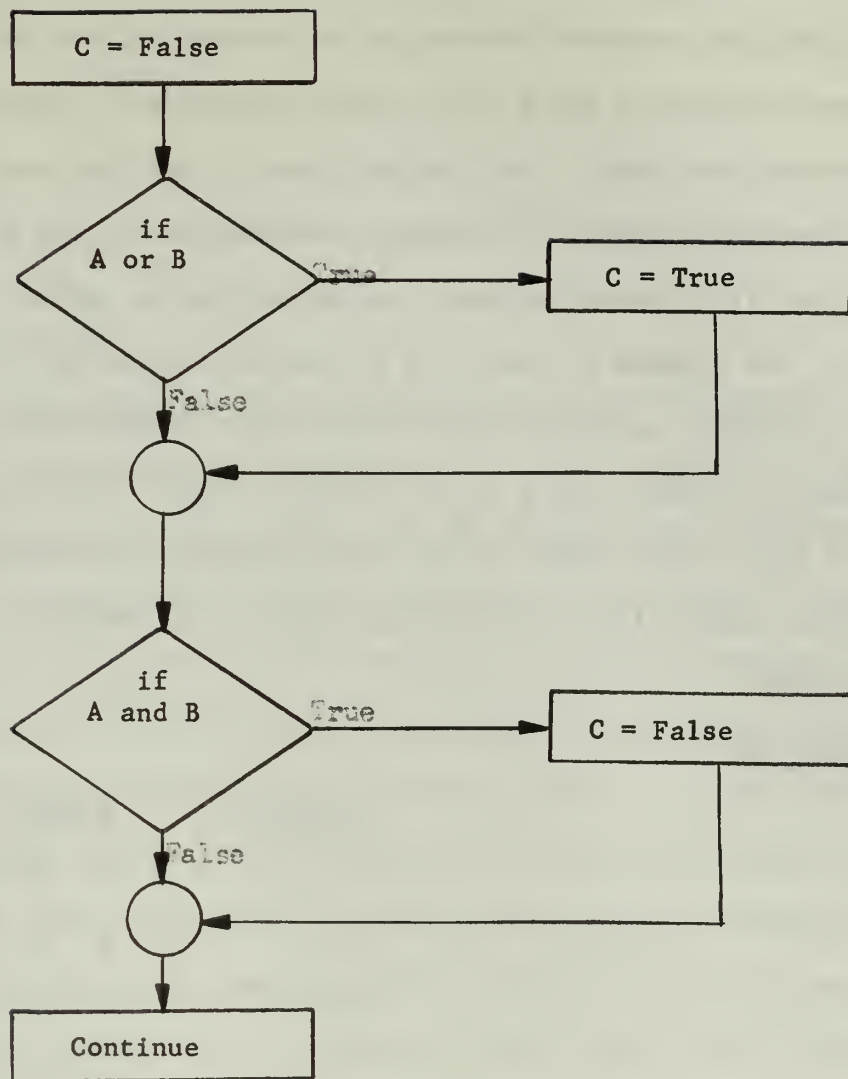
Paths are

$\{e_1\}$  ,  $\{e_2 \ e_5\}$  ,  $\{e_2 e_3 e_4\}$  .



Block Diagram of Process to Find All Paths

Figure 4.2



Block Diagram of Ringsum Operation in Computer Program

Figure 4.3

## 4.2 Path Finding Program.

A digital computer program can be devised to find all the paths between terminals x and y. The program discussed in Appendix (6) accomplishes this task. (For further details, see that section).

The generation of all possible combinations of the fundamental circuit sets is an important step, and no one may be omitted. The method used by the program to find them all is the following: to count from 4 to 7 in binary, one adds "100" to the binary numbers from 0 through 3. Similarly to count from 8 to 15, one adds "1000" to the binary numbers from 0 to 7. See example (4.2). This procedure guarantees that all possible combinations are obtained, because all numbers are obtained in the analogy.

### Example 4.3:

<u>Decimal</u>	<u>Binary</u>
0 =	0 0 0 0
1 =	0 0 0 1
2 =	0 0 1 0
3 =	0 0 1 1

to count from 4 to 7 in binary, add 100 to those numbers preceeding it.

4 =	0 1 0 0
5 =	0 1 0 1
6 =	0 1 1 0
7 =	0 1 1 1

Similarly, from 8 to 15, add 1000

8 =	1 0 0 0
9 =	1 0 0 1
10 =	1 0 1 0
11 =	1 0 1 1

<u>Decimal</u>	<u>Binary</u>
12 =	1 1 0 0
13 =	1 1 0 1
14 =	1 1 1 0
15 =	1 1 1 1

#### 4.3 Branch Loading.

When a communication net is analyzed, each link is examined individually. In addition to the reliability of the link to perform its function, there is needed a measure of the relative importance of the link in the various modes of operation of the complete net. For example, the importance of a link may be different, if complete communication is required, than if only K-terminal communication is required. One such measure of relative importance of a link is called the load of a branch.

Fu has defined the load of a branch to be the total number trees that contain this branch, [3]. As shown before, complete communication is possible only when the net contains at least a tree in the operational state. Since each branch is contained in one or more trees of the net, the number of trees that contain this branch can be found, and this number used to indicate the relative importance of the branch in complete communication. This number can be normalized, if desired, since the total load of all branches can be quickly calculated by a formula shown by Fu, [3], as

$$\sum_{i=1}^n l_i = (v - 1)t$$

where  $l_i$  ( $i = 1, 2, \dots, m$ ) is the load of branch  $b_i$  in a network with  $v$  number of vertices,  $n$  branches, and  $t$  number of trees.

This method of branch loading determination can be extended to the k-terminal communication case by means of subtrees. However, branch loading can also be defined in another way.

Definition 4.1: Branch Loading.

The load of a branch,  $b_i$ , is defined to be the total number of paths (path sets) that contain this branch.

A path set was defined to be a set of branches of a communication path between a pair of terminals. When complete communication is considered, definition (4.1) makes use of all the paths between all pairs of terminals. This means that the numbers obtained for the various branch loadings will be larger than those obtained using the definition of  $F_u$ ; however, the relative magnitudes will be the same. That is, the branch with the highest loading will be identified to be the same by either definition. The advantage of definition (4.1) lies in the simple way to evaluate rather than having to find the trees of the net.

#### 4.4 Path Length

When a communication net is analyzed, all the paths between terminal pairs are found. The loading of the branches is measured. Now the paths themselves must be examined. The length of the communication path is of great significance. We know from experience that the more often data is handled, the greater the chance for error. This is confirmed by this simple example.

Example 4.4:

Let  $p$  be the reliability of all branches,  $b_i$ , and  $p < 1$ . Then the reliability through one branch is  $p$ . The reliability of communication through two branches is  $(p \times p) = p^2$ ; through  $j$  number of branches:



$p^j$ . Since  $p < 1$ , then  $p^j < p$  for all  $j > 1$ .

This effect of path length appears directly in the 2-terminal reliability function by means of the secondary path product terms. Section (4.6) discusses the effect of the secondary path product terms. See also section (5), Application of Ideas without the Aid of a Computer.

#### 4.5 Branch Reliability.

Much work is currently being done by many investigators in the fields of Operations Research, Reliability, Non-Destructive Testing, and many others, to provide a high quality measurement of the reliability of a communication link. This work does not concern itself with the finding of this value of reliability. It must be pointed out, however, the significance of accuracy in this value and its effect on the interpretation of the results.

Results have shown that considering a net with  $\underline{e} = 10$  branches, and assuming all branch reliabilities to be equal to .90, the fourth significant figure has risen to the importance level of being the deciding factor in net configuration.

#### 4.6 Secondary Path Product Terms.

The secondary path product terms of definition (3.4) are not of secondary importance in the determination of the reliability function. As can be seen from example (3.1), the majority of nonzero canonical terms are secondary path product terms. The exact number of these terms was given in theorem (3.1), namely, for an  $A$ -edged path, there will be  $2^{(e-A)} - 1$  number of secondary path product terms.

The theorems in this section are subjected to the following conditions: A graph contains  $e$  number of edges, and  $n$  number of vertices. The branch reliability,  $p$  of branch  $b_1$ , is greater than its complement,

$\bar{p}_i$ , and is the same for all branches.

Definition 4.2: Value of a path product term.

The value of a path product term is defined to be the product of the branch reliabilities of the unbarred variables and the complement of the branch reliabilities of the barred variables of the path product term. It is applicable to both primary and secondary path product terms. It always takes on a value less than one, since it is a product of numbers each less than one.

Theorem 4.1:

Any one of the secondary path product terms has a larger value than the primary path product term associated with it, for any A-edged path,  $P_{x,y}$ .

Proof:

The value of a primary path product term is determined using definitions (3.3) and (4.2) to be

$$p^A \cdot \bar{p}^{(e-A)}$$

By definition (3.4) a secondary path product term must have at least one more unbarred variable than the primary term. Therefore, we write

$$p^{(A+a)} \cdot \bar{p}^{(e-A-a)}$$

where  $a$  is a positive integer, i.e.,  $a \geq 1$

$$\text{If } p^A \cdot \bar{p}^{(e-A)} < p^{(A+a)} \cdot \bar{p}^{(e-A-a)}$$

$$\text{Then } 1 < p^a \cdot \bar{p}^{(-a)}$$

$$\text{Since } \bar{p} < p,$$

$$\text{Thus } p^A \cdot \bar{p}^{(e-A)} < p^{(A+a)} \cdot \bar{p}^{(e-A-a)}.$$



Corollary 4.1:

The sum of the values of the primary and all the secondary path product terms of a 1-edged path is greater than that of any multi-edged path between the same terminals  $\underline{x}$  and  $\underline{y}$ .

Proof:

This Corollary follows directly from the theorem (4.1) by letting  $\underline{A}$  be equal to 1.

Theorem 4.2:

The sum of the values of all the primary and all the secondary path product terms of an  $A$ -edged path is greater than that of an  $(A+a)$ -edged path, where  $\underline{a}$  is any positive integer; i.e.,  $a \geq 1$ .

Proof:

Equation (3.5) can be written for an  $(A+a)$ -edged path.

$$\sum_{i=0}^{e-A-a} K(i) p^{(A+a+i)} \bar{p}^{(e-A-a-i)} = \binom{e-A-a}{0} p^{(A+a)} \bar{p}^{(e-A-a)} +$$
$$\binom{e-A-a}{1} p^{(A+a+1)} \bar{p}^{(e-A-a-1)} + \binom{e-A-a}{2} p^{(A+a+2)} \bar{p}^{(e-A-a-2)} + \dots (4.1)$$
$$+ \binom{e-A-a}{e-A-a-1} p^{(e-1)} \bar{p}^1 + \binom{e-A-a}{e-A-a} p^e$$

Comparing equation (4.1) to equation (3.5), we see that theorem (4.2) can be proved if it can be shown that, using definition (4.2), the value of equation (3.5) is greater than the value of equation (4.1). This can be done by showing that each coefficient of equation (4.1) is less than that of equation (3.5) for the corresponding power of  $p$ .

Consider the general term,  $i = j$ . If

$$\binom{e-A-a}{e-A-a-j} p^{(e-j)} \bar{p}^j < \binom{e-A}{a-A-j} p^{(e-j)} \bar{p}^j \quad (4.2)$$

then

$$\binom{e-A-a}{e-A-a-j} < \binom{e-A}{e-A-j} \quad (4.3)$$

then

$$\frac{(e-A-a)!}{(e-A-a-j)!(e-A-a(e-A-a-j))!} < \quad (4.4)$$

$$\frac{(e-A)!}{(e-A-j)!(e-A(e-A-j))!}$$

then

$$\frac{(e-A-a)(e-A-a-1)(e-A-a-2) \dots (e-A-a-j+1)}{j!} < \frac{(e-A)(e-A-1)(e-A-2) \dots (e-A-j+1)}{j!} \quad (4.5)$$

There will be  $j$  number of terms in the numerator of both sides of equation (4.5), each term of which is greater than one.

Since,

$$\begin{array}{ccc} e-A-a & < & e-A \\ e-A-a-1 & < & e-A-1 \\ e-A-a-2 & < & e-A-2 \\ . & & . \\ . & & . \\ . & & . \\ e-A-a-j+1 & < & e-A-j+1 \end{array}$$

Then equation (4.5) is true. From it equations (4.4), (4.3), and (4.2) follows, and the theorem is proved.

Example 4.5:

The following tables are the numerical values of the 2-terminal reliability function discussed in examples (3.1 and 2). Let the branch reliabilities be all equal and have the reliability 0.9. Columns 3,4, and 5 are the values when one branch is removed from the graph. Column 6 is the value of the 2-terminal reliability function if the specified branch has the reduced reliability as shown.

	<u>with all branches present</u>	<u>without branch #1</u>	<u>without branch #2</u>	<u>without branch #3</u>	<u>with <math>p_x=0.5</math></u>
$q_{x,y}$	.981	.81	.90	.90	.905
$q_{y,z}$	.981	.90	.81	.90	.905
$q_{x,z}$	.981	.90	.90	.81	.905

The value of  $q_n$  .972

The ideas of section (3.3) can be used to find the value of the 2-terminal reliability function in the cases of examples (3.1 and 2). Equation (3.8) is applicable in all the cases where the reliabilities of the branches are all equal; equation (3.7) where they are not all equal.

In all of these examples there exists one 1-edged path and one 2-edged path. This means, using equation (3.8),

$$\begin{aligned} \left| Q \right|_{P_1} &= r = p = .9 = .9 \\ \left| Q \right|_{P_2} &= r^2 \bar{r} = p^2 \bar{p} = (.9)(.9)(.1) = \underline{.081} \end{aligned}$$

The value of  $q_{x,y}$  is .981

Observe that this is the same result obtained in example (4.5).

Using equation (3.7) the case of example (3.1) where the reliability of branch one has the reduced reliability,  $p_1 = 0.5$ .

$$\begin{aligned} \left| Q \right|_{P_1} &= r_1 = p_1 = .5 = .5 \\ \left| Q \right|_{P_2} &= r_2 r_3 \bar{r}_1 = p_2 p_3 \bar{p}_1 = (.9)(.9)(.5) = \underline{.405} \end{aligned}$$

The value of  $q_{x,y}$  is .905

Observe that this is the same value found in example (4.5). It is significant to observe that in this example, if branch two had the reduced reliability, the effect on the value of  $q_{x,y}$  would not be as much. Consider example (3.1), let branch two assume the reduced branch reliability,  $p_2 = .5$ , and calculate the value of the 2-terminal reliability function,  $q_{x,y}$ , using equation (3.7).

$$\begin{aligned} \left| Q \right|_{P_1} &= r_1 = p_1 = .9 = .9 \\ \left| Q \right|_{P_2} &= r_2 r_3 \bar{r}_1 = p_2 p_3 \bar{p}_1 = (.5)(.9)(.1) = \underline{.045} \end{aligned}$$

The value of  $q_{x,y}$  .945

Observe that this value is higher than that one found in the case where branch one had the reduced reliability: .945 vice .905.

## 5. Application of Ideas without the Aid of a Computer

Some of the ideas presented in the preceding sections can be applied, without the benefit of a computer, in the analysis of a communication net to give rough, but fast, measurements of the properties of a net. The following subsections discuss important properties of the communication net which must be examined in the event a change in the construction of the net is required. For example, the commander of several communication systems may order that one link be removed from one system and be given to a second system. The effect on the system caused by the removal of a link can be studied qualitatively without the aid of a computer.

### 5.1 Branch loading.

If the graph of a communication net is simple or if sufficient time is available, all the paths of the net can be identified, and the loading of each branch can be found. Immediately this indicates that the branch which is used most often should not be given up, because all the paths using that branch are also given up. On the other hand, the lightest loaded branch may not be sacrificed in every case without additional examination of the topology of the graph. That branch could be the only connection to a terminal, and the destruction of this branch would isolate the terminal. However, in certain conceivable situations, the price of isolation of a terminal may be acceptable, for example, security reasons. The full effect of branch loading on the decision to remove a branch must be tempered by additional factors.

### 5.2 Path Length.

Theorem (4.2) proves that long paths are not as reliable as shorter ones. Therefore, one must place a figure of merit on the communication



path in relation to its length. The removal of either one of two branches may still enable communication between stations; however, in one case the remaining paths are longer than in the other. Then, the choice would be to remove the one that would leave the shorter path. This choice may be a poor one in light of the other factors which must be considered.

### 5.3 Branch Reliability.

Naturally the designer of a communication net will want to avoid the use of a branch with low reliability. And, if one link has to be removed, then that one of lowest reliability should be selected. However, this is not always true, especially when that branch is the only link to a certain terminal. Again the price of isolation of that terminal may be acceptable in a situation where security considerations are paramount. Likewise, the link of highest reliability may be chosen to be removed from the given net for use elsewhere, especially in cases where the branch loading is low and there exist other paths of communication of acceptable reliability.

### 5.4 Secondary Path Terms.

The importance of the secondary path product terms has been stressed before. It is pointed out in this section that since they constitute the majority of terms in the 2-terminal reliability function, and they are inversely proportional in number to the length of path generating them, the loss of a short communication path is very costly in terms of reliability.

The number of  $k$ -edged paths which will be lost when a given branch,  $b_i$ , is removed can be predicted in certain cases.

Theorem 5.1:

When branch,  $b_i$ , is removed from a graph, one 1-edged path will be removed.

Proof:

There can be only one 1-edged path between terminals  $\underline{x}$  and  $\underline{y}$  consisting of the branch  $b_i$  alone. Removal of the branch,  $b_i$ , results in the removal of that path.

Theorem 5.2:

Let  $D_x$  and  $D_y$  be the degree of terminals  $\underline{x}$  and  $\underline{y}$  respectively. Let  $b_i$  be a branch connecting terminals  $\underline{x}$  and  $\underline{y}$ . Let  $c$  be the number of additional branches connecting terminals  $\underline{x}$  and  $\underline{y}$ . Then the number of 2-edged paths between any pair of terminals, but using branch  $b_i$ , that will be removed if branch  $b_i$  is removed, is given by

$$N_2 = (D_x + D_y - 2c - 2) \quad (5.1)$$

Proof:

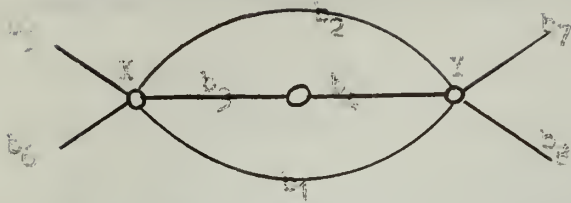
The degree of a terminal is the number of edges incident to it. A 2-edged path using the branch,  $b_i$ , must either start at terminal  $\underline{x}$  and proceed through  $\underline{y}$ , or start at terminal  $\underline{y}$  and proceed through  $\underline{x}$ . In the former case, there are  $(D_y - 1)$  number of possible 2-edged paths minus the  $c$  number which would form a circuit, since they are connected to terminal  $\underline{x}$  also. Similarly in the latter case, there are  $[(D_x - 1) - c]$  number possible. The sum is  $N_2$ :

$$N_2 = D_y - 1 - c + D_x - 1 - c$$

$$N_2 = D_y + D_x - 2c - 2$$



### Example 5.1



$$D_x = 5$$

$$D_y = 5$$

$$c = 1, \text{ namely } b_2$$

Figure 5.1

$$N_2 = 5 + 5 - 2(1) - 2 = 6$$

Namely,

$$\text{path sets } \{b_1 b_3\}, \{b_1 b_5\}, \{b_1 b_6\}, \{b_1 b_4\}, \{b_1 b_7\}, \\ \text{and } \{b_1 b_8\}$$

### Rule 5.1.

A rule which establishes a series of operations that indicates the number of 3-edged paths between any pair of terminals containing the branch,  $b_1$ , that will be removed if the branch,  $b_2$ , is removed. It is stated here without proof; however an example is given.

Step #1 Starting at node  $\underline{x}$ , proceed to the terminal of a branch incident to node  $\underline{x}$ .

Step #2 Call this node #1 having degree  $D_1$ . Let  $c$  be the number of branches having terminals  $\underline{x}$  and  $\underline{y}$ , but not being branches,  $b_1$ . Let  $d_i$  be the number of branches having node  $\underline{y}$  and node  $\underline{i}$  as terminals. Then

$$D_i + D_y - 2 - c - 2d_i = N_3 \quad (5.2)$$

Step #3 Proceed to all other nodes of the branches incident to node  $\underline{x}$ , except the node  $\underline{y}$ , of course. Repeat the calculation of equation (5.2), adding the result to the previous value of  $N_3$ .

Step #4 Proceed to the terminals of the branches incident to node y and whose terminal nodes are not those previously numbered or node x. Count the number of branches which are not incident to x or y or the nodes previously numbered in steps (2) or (3). Add this number of the previous value of  $N_3$ .  $N_3$  is the number of 3-edged paths that will be removed if branch,  $b_i$ , is removed.

Example 5.2:

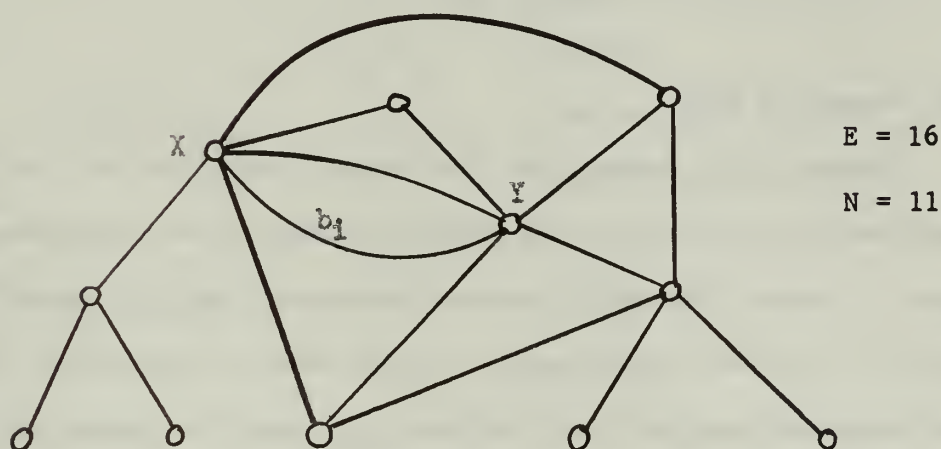


Figure 5.2

$$c = 1$$

$$D_x = 6$$

$$D_1 = 3$$

$$d_1 = 0$$

$N_3$

$$3 + 6 - 2 - 1 - 0 = \underline{6}$$

$$D_2 = 3$$

$$d_2 = 1$$

$$3 + 6 - 2 - 1 - 2(1) = \underline{4}$$

$N_3$

$$N_3 = 6 + 4 = \underline{10}$$

$$D_3 = 2, \quad d_3 = 1$$

$$2 + 6 - 2 - 1 - 2(1) = 3$$

$$N_3 \quad N_3 = 10 + 3 = \underline{\underline{13}}$$

$$D_4 = 3, \quad d_4 = 1$$

$$3 + 6 - 2 - 1 - 2(1) = 4$$

$$N_3 \quad N_3 = 13 + 4 = \underline{\underline{17}}$$

Next node 5 where 2 branches qualify

$$N_3 = 17 + 2 = \underline{\underline{19}}$$

ANS: 19

### 5.5 Number of Paths

Theorem (3.3) proves that the primary path term will not be a duplicate of any entry in the Table of Combinations regardless of the number of paths that exist between terminals x and y. Therefore, each path will contribute some value, in the sense of definition (4.2), to the 2-terminal reliability function,  $q_{x,y}$ , and its removal will diminish this value.

An examination of the paths between terminals x and y alone is not sufficient when n-terminal reliability is concerned. What may appear to be an expendable path between terminals x and y, may be a vital path between two other terminals.

While a large number of paths between the terminals of consideration indicates that the loss of one path would not cause isolation, the length of the paths may be such as to result in low reliability. All the factors mentioned previously continue to apply even when there are a large number of paths. This is also true when there are a small number of paths.

## 5.6 Addition of a branch.

The addition of one branch to a graph is equivalent to the addition of one communication link to a net. This can be viewed as a design problem in which there are eight cases to be studied:

What should the second branch terminal be, if the given objective is to:

		the first branch terminal is required to be:
maximize $q_{x,y}$	(1)	terminal <u>x</u>
	(2)	terminal <u>a</u> , not <u>x</u> or <u>y</u>
	(3)	optional
maximize $q_k$	(4)	terminal <u>a</u> in set <u>k</u>
	(5)	terminal <u>a</u> not in set <u>k</u>
	(6)	optional
maximize $q_n$	(7)	terminal <u>x</u>
	(8)	optional

These eight cases can lead to many more if the reliability of the branch to be added and the reliabilities of the other branches are permitted to change. The following discussion of the above cases assumes that all branch reliabilities are the same, and equal  $p$ .

### Case (1):

Corollary (4.1) specifies terminal y as the second branch terminal.

### Case (2):

Find all the paths between terminals x and a, and y and a. Choose terminal x or y depending on which has the fewer paths to terminal a. This will provide the most paths, with the fewest branches, between x and y.

Case (3):

Corollary (4.1) specifies the selection of terminals x and y.

Case (4):

Find all the paths between all the pairs of terminals of set k. If any one pair has an unusually small number, examine the topology of the graph in light of sections (5.1-4). Base the selection on the unusual topology. Do the same thing in the event one pair has an unusually large number of paths between them. Otherwise, locate the terminal that has the most number of paths to terminal a. Select this as the second terminal for the branch to be added. This choice will add one 1-edged path, but also the maximum number of 2- and 3-edged paths between the other terminals in the set k. This will increase the value of  $q_k$  the most.

Case (5):

Find all the paths between all the pairs of terminals of set k and terminal a. Examine the topology of the graph in light of sections (5.1-5). Especially, look for vertices of degree one or two. Unusual configurations must be solved in this manner. Otherwise, select as the second terminal that one in set k that has the largest number of paths to terminal a. See Case (4).

Case (6):

Find all the paths between all the terminal pairs in set k. Choose as the terminals of the branch to be added those between which there already exist the largest number of paths. This will not be true if an examination of the topology reveals an unusual nature to the net. See Case (4).

Case (7):

Find all the paths between all the terminals of the graph and terminal x. Examine the topology of the graph. If nothing extraordinary



appears, choose as the second terminal for the branch to be added that terminal which already has the most paths to terminal x.

Case (8):

Find all the paths between all the terminal pairs of the graph. Study the topology of the net. Choose the terminal pair between which there already exist the largest number of paths.

If there exist a large number of paths between two terminals of a graph, the paths are usually of long length. This means that they rely on many other branches, and so are of low reliability. The adding of a 1-edged path between these terminals immediately increases the 2-terminal reliability between them, and also provides an increase in the number of 2-edged paths between other pairs of terminals. In fact, it increases the number of all  $k$ -edged paths. Since there were all those paths in existence, the accessibility of the new branch by the other terminals produces a maximum number of paths between all terminals.

Lemma (3.1) shows that an isolated terminal, I, will cause both the 2- and the  $n$ -terminal reliability function to be zero. When an isolated terminal exists, clearly the topology of the graph is unusual. The isolated terminal must be selected as the second terminal for the branch to be added in each case that the terminal I is in the set k. This statement covers all the cases since the set k includes all situations from 2 to  $n$  number of terminals.

Example 5.3:

Three illustrations of a 2-terminal reliability function are discussed for the graph of figure (5.3):  $q_{af}$ ,  $q_{b,g}$ ,  $q_{c,g}$ . All the branch reliabilities are equal,  $p = .9$ .



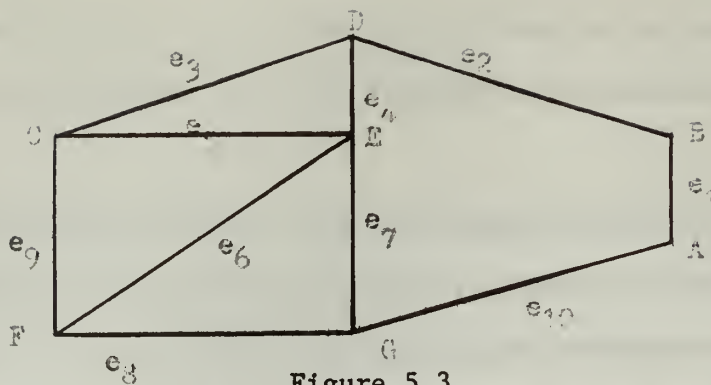


Figure 5.3

	with all branches present	without branches #3	without branches #6	without branches #9
$q_{a,f}$	.9859	.9685	.9755	.9750
$q_{b,g}$	.9940	.9621	.9922	.9897
$q_{c,g}$	.9963	.9854	.9937	.9830

Branch Loading:

	Branch number:	1	2	3	4	5	6	7	8	9	10	
$q_{a,f}$	Branch load:	6	4	4	4	4	3	5	3	4	4	10 paths
$q_{a,g}$		1	3	4	3	3	3	3	4	3	1	8 paths
$q_{c,g}$		3	4	3	4	3	4	3	3	3	3	9 paths
	Total loading:	10	11	11	11	10	10	11	10	10	8	

In the illustrations given, branch #3 is not the one to be removed.

However, branch #10 should not be removed either because this would cause node A to rely completely on branch #1 for communication. The low level of branch loading for branch #10 indicated that it is ineffectively connected, not that it can be sacrificed.

## 6. Digital Computer Program.

### 6.1 Program Specifications.

A digital computer program has been written in the FORTRAN IV Language suitable for use on the IBM System 360 computer complex. The primary purpose of the program is to compute the 2-terminal reliability function,  $q_{x,y}$ . This program is included as Appendix I. The functional block diagram, figures (6.1), (6.2) and (6.3) is fully explained in section (6.2).

The capabilities of the program go beyond the computation of the 2-terminal reliability function,  $q_{x,y}$ . After finding  $q_{x,y}$ , the program can calculate its numerical value; then, it can recalculate it under the condition that a branch of the graph has been removed, and that a branch has a different value of reliability. (See options, Section 6.1.1.) After the second and any succeeding runs, (a run is defined as the computation of one 2-terminal reliability function,)  $q_k$ , where  $k$  is the set of terminal pairs of the preceeding runs, can be computed, and its numerical value found. In finding  $q_{x,y}$ , all path sets between terminal pairs are found; this information is given as a preliminary output.

#### 6.1.1 Input

The input required by the computer program consists of three types:

(1) The number of edges, and the number of vertices of the graph are the first data required. The number of fundamental circuits is calculated from this data using the relation:

$$(e - n + 1) = w$$

Also on the first data card, the option SEL1 is provided which, if used, enables the branch reliabilities to be input. This will allow the

calculation of the numerical value of  $q_{x,y}$ , and later, the numerical value of  $q_k$ , if requested.

(2) The  $w$  number of fundamental circuits are the next input data. If the branch reliabilities are to be used, they are read in next.

(3) The data required by each run is read next. This data consists of three main parts:

- a) the terminal vertices (numbered);
- b) one path between these vertices;
- c) option selections:
  - 1) SEL - additional runs will follow
  - 2) SEL2 - find  $q_k$
  - 3) SEL4 - calculate the numerical value
  - 4) SEL5 - of  $q_{x,y}$  with the specified branch
  - 5) SEL6 - having been removed from the graph
  - 6) SEL7 - calculate  $q_{x,y}$  using the specified value of branch reliability for the specified branch

Precise details of the format for the data cards are provided in section (6.3).

#### 6.1.2 Output.

The output provided by the computer program is fully labelled for easy reading. All the input data is printed with the generated data to provide a complete record of the run. The output of generated data include:

- 1) Table of Combinations;
- 2) path matrix;
- 3) branch loading information;

4) number of k-edged paths;

5) numerical values of  $q_{x,y}$  and  $q_k$ .

At the conclusion of the computer program, a summary of the branch loading and the number of k-edged paths is printed. A message explaining the error stop is printed as necessary.

### 6.1.3 Theoretical Limitations.

In its present form, the computer program is limited to a graph containing no more than 15 edges, or 10 nodes, or 10 fundamental circuits. There must be at least one circuit in the graph. Other limitations include a maximum of 4000 entries in the Table of Combinations for all runs, and a maximum of 210 runs. The reason for these limits is the storage requirement, especially that for the Table of Combinations. Different programming techniques might be able to increase these limits slightly. The fact remains that the powers of 2 increase very rapidly, regardless of the programming technique.

### 6.2 Block Diagram.

The functional block diagram, figures (6.1,2 and 3), is presented to give a better description of the operation of the computer program. The blocks are identified in the listing of the computer program, Appendix I.

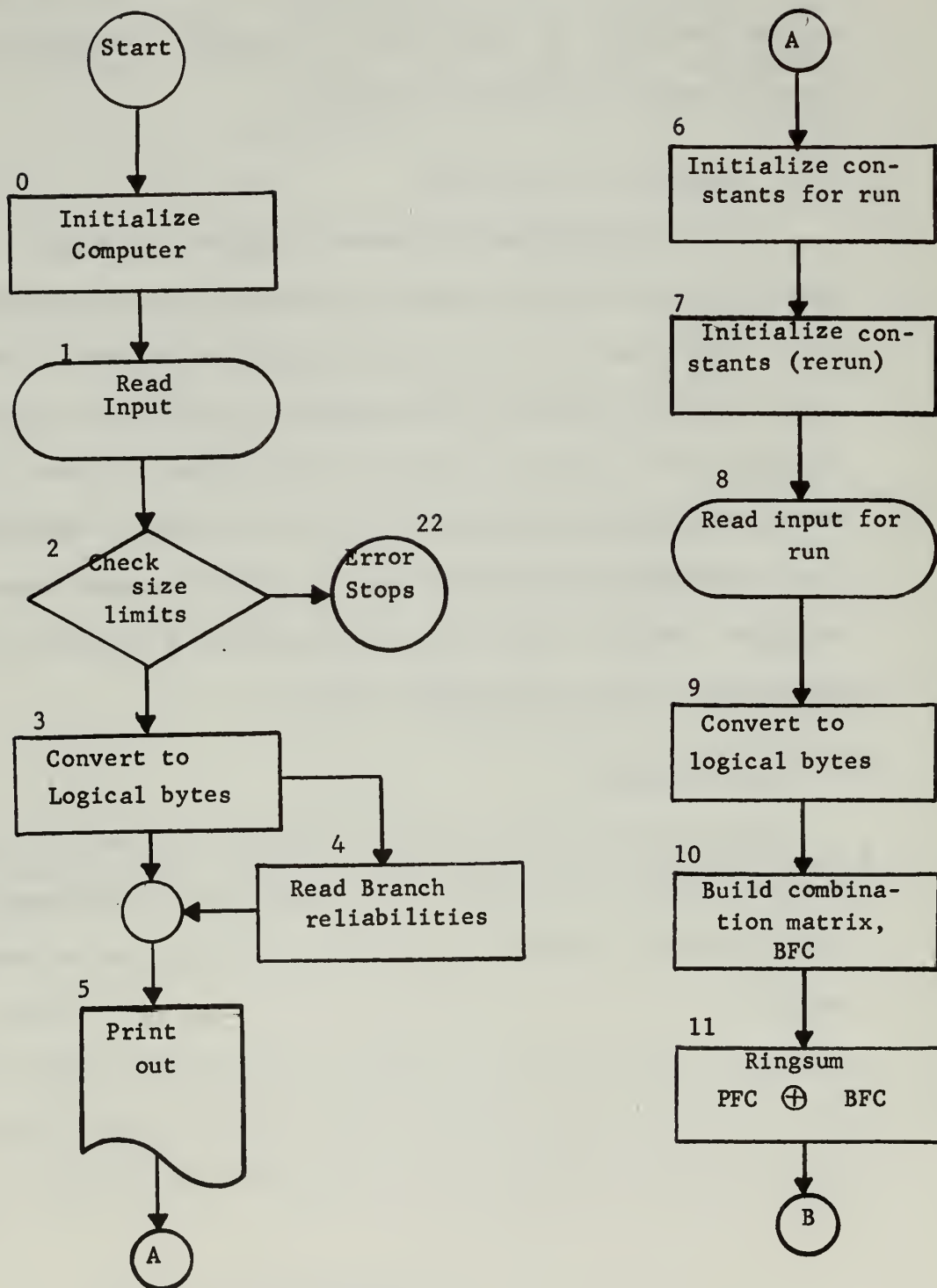


Figure 6.1



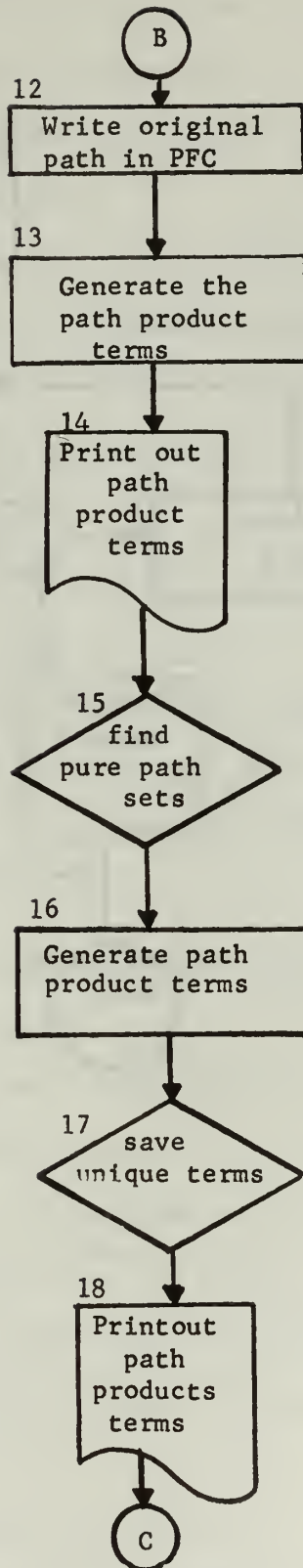


Figure 6.2



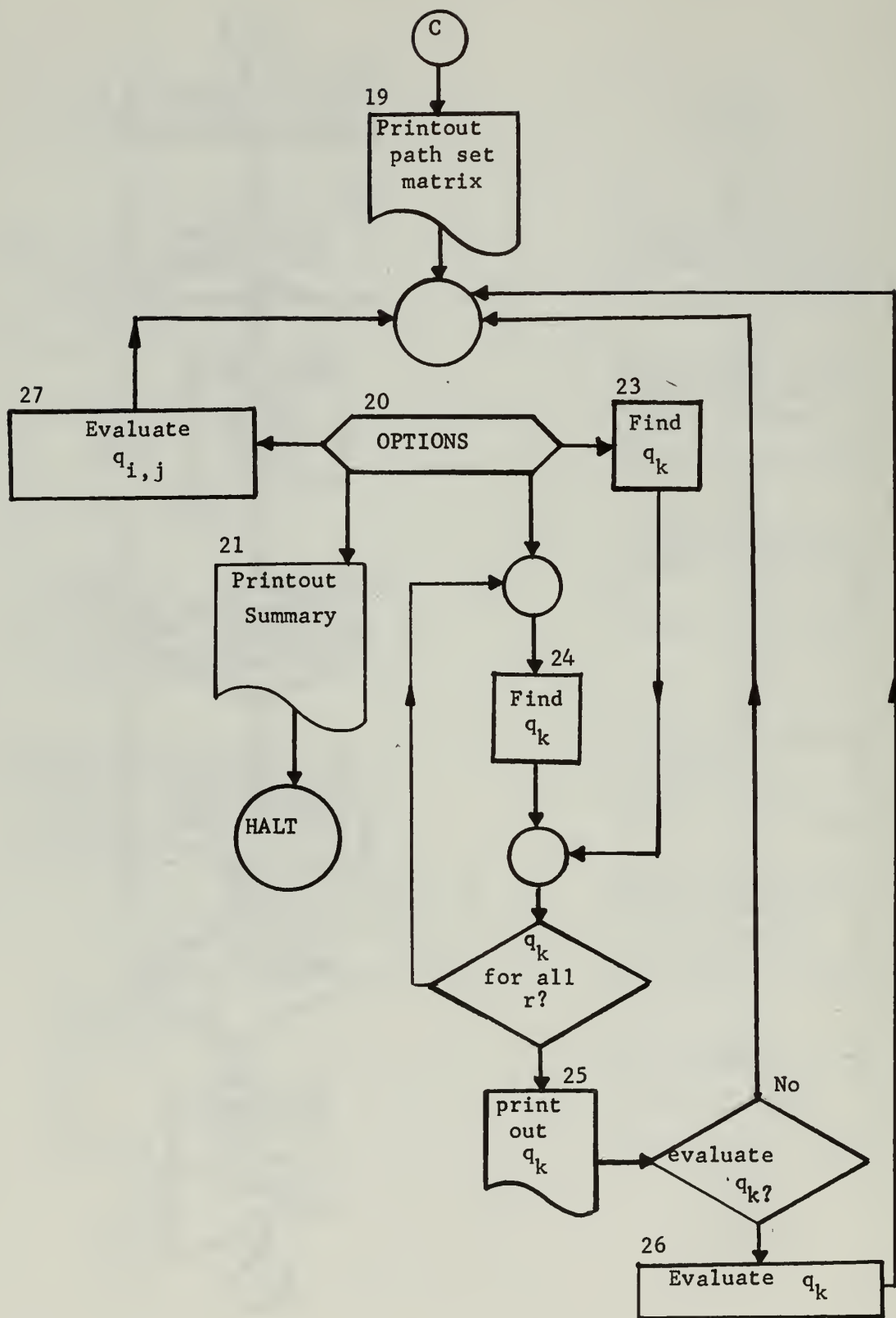


Figure 6.3

## Description of Block Diagram

Block 0: Control cards and dimension and format statements are required to prepare the computer for running.

Block 1: Input data are read including number of branches and nodes, and the fundamental circuits.

Block 2: Graph size is tested to be within program limits.

Block 3: Input data is converted to logical elements to facilitate faster calculation.

Block 4: If branch reliabilities are to be used, they are read in now into DREL matrix.

Block 5: Preliminary printouts are made for more complete data reference for the final output.

Block 6: Values required for first run of the program are set.

Block 7: Values required for succeeding runs are set.

Block 8: Data required for the run is read including terminals and path.

Block 9: Path data is converted to logical elements, and a storage area prepared for later use.

Block 10: All possible combinations of the given circuits are generated and stored in matrix BFC.

Block 11: The ringsum operation is performed on the given path and each entry in matrix BFC, and the result is written in BRP.

Block 12: The original path is written into the path matrix, PFC.

Block 13: The primary and all the secondary path product terms are generated into OPC.

Block 14: The path product terms are printed out.

Block 15: Matrix BRP is searched for pure path sets.

- Block 16: The primary and secondary path product terms are generated for the newly formed path set, and written into QTEMP.
- Block 17: Each entry in QTEMP is compared to all entries in QPC.  
If a duplicate is not found, the set from QTEMP is written in QPC.
- Block 18: Those sets added to QPC are printed out. The number of entries in QPC and the number in PFC are printed.
- Block 19: The path matrix, PFC, is printed out.
- Block 20: The various OPTIONS are examined. If  $Q_{i,j}$  or  $Q_n$  are to be found, transfer is made; if another run is to be made, transfer is accomplished.
- Block 21: Before stopping, a summary is printed out including branch loading, number of paths between each node, and path length.
- Block 22: Error stops are provided.
- Block 23: Generate matrix  $Q_n$  from path product terms common to the first 2  $q_{i,v}$  matrices.
- Block 24: Generate matrix  $Q_m$  from the previous  $Q_n$  and the newly formed  $q_{i,j}$  for this run.
- Block 25: Print out matrix  $Q_n$ .
- Block 26: If the branch reliabilities are provided, determine the value of  $Q_n$ . Transfer control to block 20 to check options.
- Block 27: If the branch reliabilities are provided, determine the value of  $q_{i,j}$ . Examine the options SEL4, SEL5, SEL6, and determine the value of  $q_{i,j}$  without the specified branch J. Examine option SEL7, and compute the value of  $q_{i,j}$  with the revised value of branch reliability. Transfer control to block 20 to check other options.

### 6.3 Data Card Configuration.

The following is a description of the data cards required for the running of the computer program as described in section (6).

Data Card		Description
<u>Type Number:</u>	<u>Columns:</u>	<u>and specifications:</u>
1	1 - 3	Number of edges, right justified; max: 15
	4 - 6	Number of nodes, right justified; max: 10
	7 - 9	If 0 or Blank: no branch reliabilities are to be used; if nonzero: branch reliabilities are to be read and used.
	10 - 72	Disregarded.
2	1 - 15	Fundamental circuit set: Up to 15 digits, 0 or 1, left justified, representing the edges in the circuit; 1 if edge present 0 if edge not present
	15 - 72	Disregarded.
	1 - 5	Blank
3	6 - 13	Decimal point and 7 digits to represent
	14 - 21	the branch reliability of the succes-
	22 - 29	sive branches 1, 2, . . . , e
	30 - 37	One field for each edge. Up to five
	38 - 45	edges per card.
	46 - 72	Disregarded.

<u>Type Number:</u>	<u>Columns:</u>	<u>Description and Specifications:</u>
4	1 - 3	Identifying number of starting node
	4 - 6	Identifying number of terminal node
	7 - 9	if 0, no more cards will be read; if not 0, additional cards read.
	10 - 12	if nonzero, $q_k$ will be computed using the previously found $q_{i,j}$ 's from all previous runs as the set $k$ ; if 0, then this is not done.
	13 - 27	left justified, up to 15 digits, 0 to 1, representing the edges in a path between the subject terminals; 0 if edge not present 1 if edge present only e number of digits processed
	28 - 30	an integer, less than or equal to e, representing an edge that has been removed from the graph. A value for $q_{i,j}$ is found assuming that edge is removed from the graph.
	31 - 33	another edge number processed as that on in the preceeding field.
	34 - 37	Same as above
	38 - 40	An edge number whose value of branch reliability it is desired to change temporarily, and recompute $q_{i,j}$ with the new value of reliability.

<u>Type Number:</u>	<u>Columns:</u>	<u>Description and Specifications:</u>
	41 - 48	A decimal point and 7 digits represent- the branch reliability of the edge specified in the preceeding field.



## 7. Conclusions.

This work shows that the reliability function as defined is an effective instrument in the analysis and design of communication nets. The theorems that are developed can be applied in many cases without the use of a digital computer to provide direction in the modification of a net. The computer can provide fast data to quantify the effect of any change to a communication net with respect to  $k$ -terminal reliability, as the results of the computer study indicate.

The computer program can be modified by specialization to facilitate sensitivity studies for branch removal or branch addition. In so doing, the limitations on graph construction can be modified. Moreover, equation (3.7) can be used in a computer program to study the 2-terminal reliability function alone.

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## APPENDIX I

The digital computer program, as described in section (6) is presented as Appendix I. The programming language is FORTRAN IV and the program is suitable for use on the IBM System 360 computer complex.

```

C
C
C      BLOCK NUMBER 0
0001      IMPLICIT LOGICAL*1(A-C,O-Q),
0002      1 INTEGER*4(E-N,R-Z)
          DIMENSION XBF(10,15),BCM(10,15),
          1BFC(1024,15),BRP(1024,15),PFC(1024,15),
          1XPFCI(210,15),XY(210,15),ENO(210,15),
          1GNO(210),QNO(400,15),QN1(400,15),
          1QTS(15),QPC(4000,15),QTEMP(400,15),
          1UNO(210),URUN(210),XPTH(15),
          1PTS(15),ST(210),SP(210),DREL(15)
C
C
0003      45 FORMAT(/,26H THE VALUE OF Q-SUB-(I,J)
0004      115H WITHOUT BRANCH113)
          46 FORMAT(/,26H THE VALUE OF Q-SUB-(I,J)
          114H WITH BRANCH 113,/,
          125H HAVING RELIABILITY VALUE1F8.6)
0005      76 FORMAT(/,6H STOP115)
0006      77 FORMAT(/,22H TOTAL BRANCH LOADING)
0007      78 FORMAT(/,25H IN PAIRS, THE NUMBER OF
          133H K-EDGED PATHS AND BRANCH LOADING)
0008      79 FORMAT(1H1,25H THE TOTAL NUMBER OF PAT
          120HHS FOUND IN EACH RUN)
0009      80 FORMAT(3I3)
0010      81 FORMAT(15I1)
0011      82 FORMAT(2I6,15I1,1I2)
0012      83 FORMAT(5X,15I3)
0013      84 FORMAT(1H1,20H THE GIVEN PATH WAS)
0014      85 FORMAT(5X,/)
0015      86 FORMAT(1H1,10H THERE ARE 1I5,
          110H LINKS AND1I5,6H NODES/)
0016      87 FORMAT(/,25H EDGE 1 2 3 4 5 6
          125H7 8 9 10 11 12 13 14 15/)
0017      88 FORMAT(/,29H THE GIVEN CIRCUIT MATRIX IS)
0018      89 FORMAT(/,20H THE PATH MATRIX IS)
0019      90 FORMAT(/,5H R=1I3,5H U=1I3)
0020      91 FORMAT(/,20H MATRIX QPC FOLLOWS)
0021      92 FORMAT(4I3,15I1,4I3,1F8.6)
0022      93 FORMAT(5X,5F8.6)
0023      94 FORMAT(/,30H THE BRANCH RELIABILITIES ARE)
0024      95 FORMAT(/,13H Q-SUB-I,J =1F10.7)
0025      96 FORMAT(/,11H Q-SUB-K =1F10.7)
0026      97 FORMAT(/,19H MATRIX QK FOLLOWS)
0027      98 FORMAT(1H1,8H RUN NO.1I6,
          16H FROM1I5,4H TO1I5,/)
0028      99 FORMAT(/,10H THERE ARE1I5,6H PATHS)

```

```

C
C
C      BLOCK  NUMBER  1,2,3,4, AND 5
0029      READ(5,80) E,N,SEL1
0030      WRITE(6,86) E,N
0031      W=E-N+1
0032      IF(E.GT.15) GO TO 3
0033      IF(E.LE.-10) GO TO 3
0034      IF(N.GT.10) GO TO 3
0035      IF(N.LE. 0) GO TO 3
0036      IF(W.GT.10) GO TO 3
0037      IF(W.LE. 0) GO TO 3
0038      DO 71 I=1,W
71      READ(5,81)(XBF(I,J),J=1,E)
0040      DO 75 I=1,W
0041      DO 70 J=1,E
0042      IF (XBF(I,J).EQ.0) BCM(I,J)=.FALSE.
0043      IF (XBF(I,J).EQ.1) BCM(I,J)=.TRUE.
0044      70 CONTINUE
0045      75 CONTINUE
0046      WRITE(6,88)
0047      DO 72 I=1,W
72      WRITE(6,83)(XBF(I,J),J=1,E)
0048      WRITE (6,85)
0049      IF (SEL1.EQ.0) GO TO 10
0050      READ(5,93)(DRFL(J),J=1,E)
0051      WRITE(6,94)
0052      WRITE(6,93)(DREL(J),J=1,E)
0053
C
C
C      BLOCK  NUMBER  6 AND 7
0054      10 TIMS=0
0055      OPT = .FALSE.
0056      OTAG =.FALSE.
0057      QTAG1=.FALSE.
0058      QTAG2=.FALSE.
0059      GR=0
0060      R=0
0061      U=0
0062      1 R=R+1
0063      URAN=U+1
0064      U=0
0065      UV=0
C
C
C      BLOCK  NUMBER  8 AND 9
0066      READ(5,92)ST(R),SP(R),SEL,SEL2,XPTH,
1 SEL4,SEL5,SEL6,SEL7,DCHG

```

```

0067      WRITE(6,84)
0068      WRITE(6,87)
0069      WRITE(6,83) XPTH
0070      WRITE(6,85)
0071      DO 54 I=1,E
0072      IF (XPTH(I).EQ.0) PTS(I)=.FALSE.
0073      IF (XPTH(I).EQ.1) PTS(I)=.TRUE.
0074      XY(R,J)=0
0075

```

C  
C  
C

```

0076      BLOCK NUMBER 10
0077      IF(OPT)GO TO 51
0078      OPT = .TRUE.
0079      DO 55 I=1,E
0080      BFC(1,I)=.FALSE.
0081      55 BFC(2,I)=BCM(1,I)
0082      K=2
0083      G=3
0084      IF(W.EQ.1) GO TO 51
0085      DO 58 I=2,W
0086      DO 57 J=1,K
0087      DO 56 F=1,E
0088      A=BFC(J,F)
0089      B=BCM(I,F)
0090      C =.FALSE.
0091      IF (A.OR.B) C=.TRUE.
0092      IF (A.AND.B) C=.FALSE.
0093      BFC(G,F) = C
0094      56 CONTINUE
0095      G=G+1
0096      57 CONTINUE
0097      58 K=G-1

```

C  
C  
C

```

0097      BLOCK NUMBER 11
0098      51 T=GR+1
0099      DO 38 I=1,K
0100      DO 37 J=1,E
0101      A=BFC(I,J)
0102      52 B=PTS(J)
0103      C =.FALSE.
0104      IF (A.OR.B) C=.TRUE.
0105      IF (A.AND.B) C=.FALSE.
0106      37 BRP(I,J)=C
0107      38 CONTINUE

```

C  
C

BLOCK NUMBER 12, 13, AND 14



```

C
0107      U=URAN
0108      UV=U
0109      DO 103 J=1,E
0110      QTS(J)=PTS(J)
0111      QPC(U,J)=PTS(J)
0112      103 PFC(T,J)=PTS(J)
0113      DO 115 J=1,E
0114      IF(QTS(J)) GO TO 115
0115      QTS(J)=.TRUE.
0116      USTOP=UV
0117      DO 102 V=U,USTOP
0118      UV=UV+1
0119      DO 101 JJ=1,E
0120      C=.FALSE.
0121      A=QPC(V,JJ)
0122      B=QTS(JJ)
0123      IF(A.OR.B) C=.TRUE.
0124      QPC(UV,JJ)=C
0125      101 CONTINUE
0126      102 CONTINUE
0127      QTS(J)=.FALSE.
0128      115 CONTINUE
0129      U=UV
0130      WRITE(6,91)
0131      WRITE(6,87)
0132      H=0
0133      DO 225 HQ=UPAN,UV
0134      H=H+1
0135      DO 224 J=1,E
0136      XPFCI(H,J)=1
0137      IF(QPC(HQ,J)) GO TO 224
0138      XPFCI(H,J)=0
0139      224 CONTINUE
0140      WRITE(6,83)(XPFCI(H,J),J=1,E)
0141      225 CONTINUE

C
C
C      BLOCK   NUMBER   15

0142      DO 228 I=1,K
0143      AD=.FALSE.
0144      DO 226 J=1,E
0145      IF(PTS(J)) GO TO 230
0146      226 CONTINUE
0147      IF(AD) GO TO 228
0148      GO TO 236
0149      230 IF(BRP(I,J)) GO TO 234
0150      GO TO 236

```

```

0151      234 AD=.TRUE.
0152      GO TO 226
C CIRCUIT EXISTENCE TEST
0153      236 AD=.FALSE.
0154      DO 299 L=1,K
0155      AD=.FALSE.
0156      DO 295 J=1,E
0157      IF(BFC(L,J)) GO TO 298
0158      295 CONTINUE
0159      IF (AD) GO TO 228
0160      GO TO 300
0161      238 AD=.TRUE.
0162      GO TO 295
0163      298 IF (BRP(I,J)) GO TO 238
0164      300 CONTINUE
0165      299 CONTINUE

C
C BLOCK NUMBER 16, 17 AND 18
C
0166      T=T+1
0167      V=1
0168      DO 104 J=1,E
0169      QTS(J)=BRP(I,J)
0170      QTEMP(1,J)=BRP(I,J)
0171      104 PFC(T,J)=BRP(I,J)
0172      DO 108 J=1,E
0173      IF(QTS(J)) GO TO 108
0174      QTS(J)=.TRUE.
0175      STEP =V
0176      DO 106 IV=1,STEP
0177      V=V+1
0178      DO 105 JJ=1,E
0179      C=.FALSE.
0180      A=QTEMP(IV,JJ)
0181      B=QTS(JJ)
0182      IF(A.OR.B) C=.TRUE.
0183      QTEMP(V,JJ)=C
0184      105 CONTINUE
0185      106 CONTINUE
0186      QTS(J)=.FALSE.
0187      108 CONTINUE
0188      KTR=0
0189      DO 113 H=1,V
0190      DO 111 IV=URAN,U
0191      DO 110 J=1,E
0192      A=QTEMP(H,J)
0193      B=QPC(IV,J)
0194      IF(A.AND.B) GO TO 110

```

```

0195      IF(A.OR.8)      GO TO 111
0196      110 CONTINUE
0197      GO TO 113
0198      111 CONTINUE
0199      KTR=KTR+1
0200      UV=UV+1
0201      DO 112 J=1,E
0202      XPFCI(1,J)=1
0203      IF(QTEMP(H,J)) GO TO 112
0204      XPFCI(1,J)=0
0205      112 QPC(UV,J)=QTEMP(H,J)
0206      WRITE(6,83)(XPFCI(1,J),J=1,E)
0207      113 CONTINUE
0208      U=UV
0209      IF(U.GT.4000) GO TO 5
0210      228 CONTINUE
0211      TNO=T-GR
0212      WRITE(6,85)
0213      WRITE(6,99) TNO
0214      WRITE(6,89)
0215      WRITE(6,87)

C
C
C      BLOCK NUMBER 19

0216      TSTR = GR + 1
0217      H=0
0218      DO 127 I =TSTR,T
0219      H=H+1
0220      X=0
0221      DO 126 J=1,E
0222      XPFCI(H,J) =1
0223      IF(PFC(I,J)) GO TO 128
0224      XPFCI(H,J) = 0
0225      126 CONTINUE
0226      WRITE (6,83) (XPFCI(H,J),J=1,E)
0227      XY(R,X) =XY(R,X)+1
0228      127 CONTINUE
0229      WRITE(6,78)
0230      WRITE(6,87)
0231      WRITE(6,83)(XY(R,X),X=1,E)
0232      WRITE(6,83)(END(R,J), J=1,E)
0233      WRITE (6,85)
0234      URUN(R)=U
0235      UNO(R)=U-URAN+1
0236      GNO(R)=TNO
0237      GR=T

C
C      BLOCK NUMBER 20, 21 AND 22

```

```

0238      C      IF(SEL1.NE.0) GO TO 15
0239      119 IF(OTAG) GO TO 120
0240      OTAG=.TRUE.
0241      121 IF (SEL.EQ.0) GO TO 53
0242      GO TO 1
0243      128 ENO(R,J)=ENO(R,J)+1
0244      X=X+1
0245      GO TO 126
0246      53 WRITE(6,79)
0247      WRITE(6,83)(GNO(I),I=1,R)
0248      WRITE(6,78)
0249      DO 6 J=1,E
0250      6 UNO(J)=0
0251      DO 2 I=1,R
0252      WRITE(6,98) I,ST(I),SP(I)
0253      DO 7 J=1,E
0254      7 UNO(J)=UNO(J)+ENO(I,J)
0255      WRITE(6,83)(XY(I,J),J=1,E)
0256      WRITE(6,83)(ENO(I,J),J=1,E)
0257      2 WRITE(6,85)
0258      WRITE(6,77)
0259      WRITE(6,83)(UNO(J),J=1,E)
0260      9 STOP
0261      3 RESTP=99
0262      GO TO 8
0263      4 RESTP=999
0264      GO TO 8
0265      5 RESTP=888
0266      8 WRITE(6,76) RESTP
0267      GO TO 9

```

C  
C  
C

BLOCK NUMBER 23

```

0268      120 IF (SEL2.EQ.0) GO TO 121
0269      IF(OTAG1) GO TO 150
0270      OTAG1=.TRUE.
0271      Z=1
0272      ZA1=UNO(1)
0273      ZA2=ZA1+1
0274      ZA3=ZA1+UNO(2)
0275      DO 125 YZ=1,ZA1
0276      DO 124 YY=ZA2,ZA3
0277      DO 123 J=1,E
0278      A=QPC(YZ,J)
0279      B=QPC(YY,J)
0280      IF(A.AND.8) GO TO 122
0281      IF (A.OR.8) GO TO 124

```

0282	122	QNO(Z,J)=A
0283	123	CONTINUE
0284		Z=Z+1
0285	124	CONTINUE
0286	125	CONTINUE
0287		YR=2
0288		GO TO 161

	C	
	C	
	C	BLOCK NUMBER 24 AND 25

0289	150	ZSTR=URUN(YR)+1
0290		YR=YR+1
0291		ZMX=Z-1
0292		ZAR=URUN(YR)
0293		Z=1
0294		DO 160 YZ=ZSTR,ZAR
0295		DO 159 YY=1,ZMX
0296		DO 158 J=1,E
0297		A=QPC(YZ,J)
0298		IF(QTAG2) GO TO 154
0299		B=QNO(YY,J)
0300	151	IF(A.AND.B) GO TO 152
0301		IF(A.OR.B) GO TO 159
0302	152	IF(QTAG2) GO TO 156
0303		QN1(Z,J)=A
0304		GO TO 158
0305	154	B=QN1(YY,J)
0306		GO TO 151
0307	156	QNO(Z,J)=A
0308		GO TO 158
0309	158	CONTINUE
0310		Z=Z+1
0311	159	CONTINUE
0312	160	CONTINUE
0313		QTAG2=.NOT.QTAG2
0314	161	IF(YR.EQ.R) GO TO 162
0315		GO TO 150
0316	162	ZMX=Z-1
0317		WRITE(6,97)
0318		DO 27 YD=1,ZMX
0319		DO 26 J=1,E
0320		XPFCI(1,J)=1
0321		IF(QTAG2) GO TO 24
0322		IF(QNO(YD,J)) GO TO 26
0323	25	XPFCI(1,J)=0
0324	26	CONTINUE
0325		WRITE(6,83)(XPFCI(1,J),J=1,E)
0326	27	CONTINUE

0327  
0328  
0329

GO TO 164  
24 IF(QN1(YD,J)) GO TO 26  
GO TO 25

C  
C  
C

BLOCK NUMBER 26

0330  
0331  
0332  
0333  
0334  
0335  
0336  
0337  
0338  
0339  
0340  
0341  
0342  
0343  
0344  
0345  
0346  
0347

164 IF(SEL2.NE.2) GO TO 121  
ZMX=Z-1  
DSUM=0.0  
DO 168 YD=1,ZMX  
DTEMP=1.0  
DO 167 J=1,E  
IF(QTAG2) GO TO 165  
IF(QN0(YD,J)) GOTO 163  
166 DTEMP=DTEMP\*(1.0-DREL(J))  
167 CONTINUE  
DSUM=DSUM+DTEMP  
168 CONTINUE  
WRITE(6,96) DSUM  
GO TO 121  
163 DTEMP=DTEMP\*DREL(J)  
GO TO 167  
165 IF(QN1(YD,J)) GO TO 163  
GO TO 166

C  
C  
C

BLOCK NUMBER 27

0348  
0349  
0350  
0351  
0352  
0353  
0354  
0355  
0356  
0357  
0358  
0359  
0360  
0361  
0362  
0363  
0364  
0365  
0366  
0367  
0368

14 DTEMP=DTEMP\*DREL(J)  
GO TO 16  
15 DSUM=0.0  
USTP=URUN(R)  
DO 17 UD=URAN,USTP  
DTEMP=1.0  
DO 16 J=1,E  
IF(QPC(UD,J)) GO TO 14  
DTEMP=DTEMP\*(1.0-DREL(J))  
16 CONTINUE  
DSUM=DSUM+DTEMP  
17 CONTINUE  
WRITE(6,98) R,ST(R), SP(R)  
WRITE(6,95) DSUM  
WRITE(6,85)  
IF(SEL4.LE.0) GO TO 31  
SELX=SEL4  
30 IF(SELX.GT.15) GO TO 3  
DTEM1=DREL(SELX)  
DREL(SELX)=0.0  
DSUM=0.0



```

0369      DO 32 UD=URAN,USTP
0370      IF(QPC(UD,SELX)) GO TO 32
0371      DTEMP=1.0
0372      DO 33 J=1,E
0373      IF(QPC(UD,J)) GO TO 36
0374      DTEMP=DTEMP*(1.0-DREL(J))
0375      33 CONTINUE
0376      DSUM=DSUM+DTEMP
0377      32 CONTINUE
0378      WRITE(6,45) SELX
0379      WRITE(6,95) DSUM
0380      WRITE(6,85)
0381      DREL(SELX)=DTEM1
0382      31 IF(SEL5.LE.0) GO TO 34
0383      SELX=SEL5
0384      SEL5=0
0385      GO TO 30
0386      36 DTEMP=DTEMP*DREL(J)
0387      GO TO 33
0388      34 IF(SFL6.LE.0) GO TO 35
0389      SELX=SEL6
0390      SEL6=0
0391      GO TO 30
0392      35 IF(SEL7.LE.0) GO TO 119
0393      IF(SEL7.GT.15) GO TO 3
0394      DTEM1=DREL(SEL7)
0395      DREL(SEL7)=DCHG
0396      DSUM=0.0
0397      DO 44 UD=URAN,USTP
0398      DTEMP=1.0
0399      DO 43 J=1,E
0400      IF(QPC(UD,J)) GO TO 42
0401      DTEMP=DTEMP*(1.0-DREL(J))
0402      43 CONTINUE
0403      DSUM=DSUM+DTEMP
0404      44 CONTINUE
0405      WRITE(6,46) SEL7,DCHG
0406      WRITE(6,95) DSUM
0407      WRITE(6,85)
0408      DREL(SEL7)=DTEM1
0409      GO TO 119
0410      42 DTEMP=DTEMP*DREL(J)
0411      GO TO 43

```

C  
C

```

0412      END

```

C GR = TOTAL NO OF PATHS AT START OF RUN  
 C T IS USED AS A RUNNING INDEX  
 C GNO(R) IS USED AS A TOTAL FOR THE RUN  
 C IF SEL IS ZERO, COMPUTER WILL HALT AFTER  
 C THIS RUN; IF NONZERO, THEN AN ADDITIONAL  
 C DATA CARD WILL BE READ AFTER THIS RUN  
 C IF SEL2 IS ZERO, NO Q-SUB-K FUNCTION  
 C WILL BE FOUND; IF NONZERO, THEN Q-SUB-N  
 C WILL BE DETERMINED  
 C NOW COMPUTE Q-SUB-N  
 C UND(R) CONTAINS LENGTH OF Q-SUB-I,J(P)  
 C R IS THE RUN NUMBER  
 C ST IS THE STARTING TERMINAL  
 C SP IS THE ENDING TERMINAL  
 C W IS THE NO. OF INDEPENDENT CIRCUITS  
 C K IS THE NO. OF POSSIBLE COMBINATIONS  
 C OF THE GIVEN CIRCUITS  
 C GR IS USED AS A RUN-TOTAL-ER  
 C STOP 99 MEANS THE NUMBER OF NODES, EDGES, OR CIRCUITS  
 C IS GREATER THAN THE LIMITS OF 15, 10, AND 10  
 C STOP 999 MEANS THE LIMIT IF Q-SUB-K HAS BEEN REACHED  
 C STOP 888 MEANS TOO MANY TERMS HAVE BEEN PRODUCED  
 C AND ALL RESULTS ARE NOW IN DOUBT  
 C SEL IS THE SELECTION CODE  
 C IF SEL1 = 0, NO BRANCH RELIABILITIES ARE READ  
 C IF SEL1 NOT ZERO, THE VALUE FOR Q-SUB-I,J IS FOUND  
 C IF SEL2 = 2, THE VALUE FOR Q-SUB-N IS FOUND  
 C MATRIX XBF CONTAINS THE INDEPENDENT CIRCUITS  
 C MATRIX BCM IS XBF AS LOGICAL ELEMENTS  
 C MATRIX XPTH CONTAINS THE GIVEN PATH SET  
 C MATRIX BFC CONTAINS ALL POSSIBLE COMBIN-  
 C ATIONS OF BCM  
 C MATRIX BRP CONTAINS THE RINGSUM OF THE  
 C PATH AND BFC  
 C MATRIX PFC IS THE PATH MATRIX, INDEXED  
 C BY T OVER ALL RUNS  
 C MATRIX XPFCI IS THE INTEGER VERSION OF PFC  
 C MATRIX XY COUNTS THE NUMBER OF K-EDGED PATHS  
 C MATRIX ENO CONTAINS BRANCH LOADING  
 C MATRIX GNO HAS THE NUMBER OF PATHS  
 C MATRIX  
 C THIS ADDS UNIQUE PATH PRODUCT TERMS  
 C TO THE COLLECTION NOW IN QPC  
 C BUILD THE PATH PRODUCT TERMS  
 C STORE FOR LATER TESTING OF UNIQUENESS

## APPENDIX II

The problems solved in examples (3.1-3) and (4.5) are combined and solved by the digital computer program. The output of the running of the program is included as Appendix II.

THERE ARE 3 LINKS AND 3 NODES

THE GIVEN CIRCUIT MATRIX IS

1 1 1

THE BRANCH RELIABILITIES ARE  
0.9000000 0.9000000 0.9000000

THE GIVEN PATH WAS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MATRIX QPC FOLLOWS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	0												
	1	1	0												
	1	0	1												
	1	1	1												
	0	1	1												

THERE ARE 2 PATHS

THE PATH MATRIX IS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	0												
	0	1	1												

IN PAIRS, THE NUMBER OF K-EDGED PATHS AND BRANCH LOADING

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	1	0												
	1	1	1												

RUN NO. 1 FROM 1 TO 2

Q-SUB-I,J = 0.9809998

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 1

Q-SUB-I,J = 0.8099999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 2

Q-SUB-I,J = 0.8999999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 3

Q-SUB-I,J = 0.8999999

THE VALUE OF Q-SUB-(I,J) WITH BRANCH 1  
HAVING RELIABILITY VALUE 0.500000

Q-SUB-I,J = 0.9049999



THE GIVEN PATH WAS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

MATRIX QPC FOLLOWS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	1	0												
	1	1	0												
	0	1	1												
	1	1	1												
	1	0	1												

THERE ARE 2 PATHS

THE PATH MATRIX IS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	1	0												
	1	0	1												

IN PAIRS, THE NUMBER OF K-EDGED PATHS AND BRANCH LOADING

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	1	0												
	1	1	1												

RUN NO. 2 FROM 1 TO 3

Q-SUB-I,J = 0.9309998

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 1

Q-SUB-I,J = 0.8999999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 2

Q-SUB-I,J = 0.8099999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 3

Q-SUB-I,J = 0.3999999

THE VALUE OF Q-SUB-(I,J) WITH BRANCH 2  
HAVING RELIABILITY VALUE 0.500000

Q-SUB-I,J = 0.9049999

MATRIX QK FOLLOWS

1	1	0
1	0	1
1	1	1
0	1	1

Q-SUB-K = 0.9719998

THE GIVEN PATH WAS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

MATRIX QPC FOLLOWS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	0	1												
	1	0	1												
	0	1	1												
	1	1	1												
	1	1	0												

THERE ARE 2 PATHS

THE PATH MATRIX IS

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	0	1												
	1	1	0												

IN PAIRS, THE NUMBER OF K-EDGED PATHS AND BRANCH LOADING

EDGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	1	0												
	1	1	1												

RUN NO. 3 FROM 2 TO 3

Q-SUB-I,J = 0.9809998

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 1

Q-SUB-I,J = 0.8999999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 2

Q-SUB-I,J = 0.8999999

THE VALUE OF Q-SUB-(I,J) WITHOUT BRANCH 3

Q-SUB-I,J = 0.8099999

THE VALUE OF Q-SUB-(I,J) WITH BRANCH 3  
HAVING RELIABILITY VALUE 0.500000

Q-SUB-I,J = 0.9049999

MATRIX OK FOLLOWS

1	0	1
0	1	1
1	1	1
1	1	0

Q-SUB-K = 0.9719998

THE TOTAL NUMBER OF PATHS FOUND IN EACH RUN

2 2 2

IN PAIRS, THE NUMBER OF K-EDGED PATHS AND BRANCH LOADING

RUN NO. 1 FROM 1 TO 2

1 1 0  
1 1 1

RUN NO. 2 FROM 1 TO 3

1 1 0  
1 1 1

RUN NO. 3 FROM 2 TO 3

1 1 0  
1 1 1

TOTAL BRANCH LOADING

3 3 3

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## 13. ABSTRACT

Topological methods may be employed in an effective manner in the analysis and design of a communication net. One application is in the determination of the reliability of the net with respect to both complete communication and k-terminal communication. The effect on this reliability caused by the removal of one link is studied.

The k-terminal reliability function is defined and its application to analysis and design of communication nets is demonstrated. A digital computer program is presented, and examples of its use are included. A formula is given to find the value of the 2-terminal function.

14.

## KEY WORDS

Communication

Reliability

Topology

Nets

## LINK A

## LINK B

## LINK C

ROLE

WT

ROLE

WT

ROLE

WT













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